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多重有序中介因子之機制分析—平均因果效應與族群介入效應

Mechanism analysis with ordered multiple mediators—average causal effect and population intervention effect

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中文摘要

過去二十年間，因果效應的機制研究中，中介效應與交互作用是核心的研究議題。學者 VanderWeele 提出的單一中介因子四路徑拆解法，則是為中介效應與交互作用的研究奠定基礎的理論框架。然而，當其他學者試圖將單一中介因子拓展至多重中介因子的時候，卻面臨諸多挑戰，因為因果效應的路徑數量隨著中介因子數目增加呈現超越指數的增長。依據本研究室 Liao 提出的雙有序中介因子二十八路徑拆解法，便呈現此問題的複雜性，雖然當初二十八路徑拆解法並未完整的引入反事實模型的定義、辨識假設、結果，但也為雙有序中介因子的機制分析提供了研究的方向。因此，本文將回顧本研究室 Liao 提出的雙有序中介因子二十八路徑拆解法，利用反事實模型定義此二十八個路徑特定效應，並且確立辨識假設以及呈現辨識結果，並與學者 Miles 提出的「中介效應準則」進行比較，仔細的討論在雙有序中介因子二十八路徑拆解法是否滿足過去主流學界對於中介效應的合理性。

此外，本文也進一步將雙有序中介因子二十八路徑拆解法拓展至族群介入效應(population intervention effect, PIE)的尺度。族群介入效應和族群可歸因分率(population attributable fraction, PAF)，在數學公式上有著相近的關係，上述的 PIE、PAF 的尺度都是公共衛生研究評估移除暴露因子對於整體族群的重要指標。而族群介入效應的尺度下，則會發展成雙有序中介因子三十三路徑拆解法，同樣的呈現其反事實模型以及辨識假設、結果，並且比較過往學者提出的 PIE、PAF 拆解法，期望為公共衛生研究的統計指標提供完整的理論詮釋。

關鍵字：平均因果效應、族群介入效應、中介效應、交互作用

Abstract

Over the past two decades, mediation effects and interaction have been central research topics in the study of causal mechanisms. VanderWeele's 4-way decomposition for a single mediator has established a foundational theoretical framework for research on mediation and interaction. However, when other scholars attempted to extend the single-mediator model to multiple mediators, they encountered challenges, as the number of causal pathways grows exponentially with the number of mediators. The 28-way decomposition method for two ordered mediators proposed by Liao in our research lab demonstrates the complexity of this issue. Although the original 28-way decomposition method did not fully incorporate the definitions, identification assumptions, or results based on the counterfactual model, it provided a direction for mechanistic analysis involving two ordered mediators. Therefore, this paper reviews the 28-way decomposition method for two ordered mediators proposed by Liao, defines these 28 path way-specific effects using the counterfactual model, establishes identification assumptions, and presents identification results. Additionally, we compare this method with the "mediation criteria" proposed by Miles, carefully examining whether the 28-way decomposition method for two ordered mediators satisfies the mainstream academic criteria for reasonable mediation analysis. Furthermore, this paper extends the 28-way decomposition method for two ordered mediators to the scale of the population intervention effect (PIE). The population intervention effect and the population attributable fraction (PAF) are mathematically related and serve as key indicators in public health research for assessing the impact of removing an exposure factor on the overall population. Under the population intervention effect framework, this method evolves into a 33-way decomposition method for two ordered mediators. Similarly, we present its counterfactual model, identification assumptions, and results, and compare it with previous PIE and PAF decomposition methods proposed by other scholars. This study aims to provide a comprehensive theoretical interpretation of statistical indicators in public health research.

Keywords：average causal effect , population intervention effect , mediation effect、interaction effect

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第一章、緒論

因果推論在公共衛生、社會科學等領域具有廣泛的應用，幫助研究者理解變數之間的因果關係，進一步為政策制定與干預措施提供依據。其中，中介效應與交互作用分析是因果推論的重要分支，旨在探討影響結果變量的機制，解析暴露因子透過中介因子產生的效應，以及交互作用如何影響因果關係。隨著因果推論方法的發展，研究者提出了不同的效應拆解方法，學者(VanderWeele, 2014)針對單一中介因子的四路徑拆解法，為中介效應與交互作用的研究奠定了理論基礎。然而，在多重中介因子的條件下，並且各中介因子間存在有序性時，傳統的分析框架難以充分描述因果效應的傳遞機制，現有方法可能低估其中的複雜性，因此，如何發展更細緻的拆解方法，完整刻畫雙有序中介因子的因果效應，成為當前因果推論研究的重要課題。

本研究聚焦於雙有序中介因子的中介效應與交互作用分析，旨在建立一個更精細的拆解框架，以更準確地刻畫因果效應的傳遞機制。為此，本文將延續過去本研究室(Liao, 2021)所提出的雙有序中介因子二十八路徑拆解法，該方法在平均因果效應(average causal effect, ACE)的基礎上，對於二十八條路徑效應的因果圖與效應詮釋提供了詳盡的分析。然而，當時的理論架構尚不完備，未能透過因果推論的反事實模型進行數學式表達與效應辨識，限制了其在因果推論分析方法的框架。因此，本研究進一步補足理論基礎，結合反事實模型進行數學推導與效應辨識，以確保結果的可解釋性與數學嚴謹性，並進一步細緻分解中介效應與交互作用的影響，提供更全面的因果推論架構。並將二十八路徑拆解法平均因果效應的尺度，拓展至族群介入效應(population intervention effect, PIE)的尺度，發展出 PIE 三十三路徑拆解法。使用 PIE 尺度的優點在於：族群介入效應(population intervention effect, PIE) 與族群可歸因分率(population attributable fraction, PAF)密切相關，這些都是評估暴露因子對公共衛生影響的因果尺度，有助於評估從族群中消除暴露因子的潛在影響。

本論文的架構如下：第二章介紹雙中介因子的平均因果效應拆解方法，回顧並在此基礎上提出更細緻的效應分解與辨識框架，並且與過往的中介效應準則進行比較，重新審視雙有序中介因子中介效應的分類方法。第三章則將雙有序中介因子的分析應用於族群介入效應，並發展 PIE 尺度的三十三路徑拆解方法，並比較過往的拆解方法。最後於第四章結論，討論本研究在未來改進之處。

第二章、雙中介因子之平均因果效應的機制分解

中介效應與交互作用分析的研究中，(VanderWeele, 2014)提出的 4-way 拆解法為單一中介因子的中介效應與交互作用分析提供了一個完整的理論框架。該方法是單一中介情境下最精細的效應拆解方式，統一了所有單一中介因子之拆解法，並且將過往的中介效應與交互作用分析中的拆解結果視為 4-way 拆解法下四個效應的組合，分別為：僅由中介效應產生的效應(pure indirect effect, PIE)、僅由交互作用產生的效應(reference interaction, INT_{ref})、同時由中介效應與交互作用效應產生的效應(mediated interaction, INT_{med})、非中介效應也非交互作用所產生的效應(contorled direct effect, CDE)。這些效應的拆解皆是建立在因果推論的框架下，透過比較不同處理組間的結果來估計平均因果效應(average causal effect, ACE)，即處理對結果變數的平均影響。在中介模型中，ACE 可進一步拆解為直接效應與間接效應，其中間接效應涉及中介變數的作用，而交互作用則強調不同變數間的相互影響。然而，多重中介因子的中介效應與交互作用分析比單一中介因子更加複雜。

時至今日，僅有兩篇文獻針對多重中介因子提出分析框架，分別為(Bellavia, Valeri, 2018)在雙平行中介因子條件下提出的 10-way 拆解法，以及(Gao et al. 2022)沿用 10-way 拆解法框架進一步發展的雙有序中介因子拆解法，總計拆解出十條路徑。

儘管如此，這些方法均低估了多重中介因子分析的複雜程度。基於此，本章將介紹本研究室(Liao, 2021)所發展的二十八路徑拆解法的最細部拆解，詳細說明其對因果參數的分解方式，並進一步探討因果推論中的辨識假設及其對效應辨識的影響，呈現二十八路徑拆解法下的辨識結果。此外，將系統地比較二十八路徑拆解法與傳統雙中介因子拆解法，分析兩者在效應分解與解釋上的異同，並針對傳統雙中介因子的效應準則分類提出質疑，期望為多重中介因子分析提供實用且可擴展的工具。

2.1 因果參數、假設與辨識

(2.1.1) 定義因果參數

本節(2.1.1)聚焦於雙有序中介因子情境下的因果參數定義，依據本研究室(Liao, 2021)提出的二十八路徑拆解方法，我們定義了六個因果參數，藉此描述雙有序中介因子所可能呈現的效應路徑。這些參數以巢狀反事實模型

(nested counterfactual model)為基礎，涵蓋暴露因子A與中介因子M_1、M_2與結果變量Y的各種組合情境，並以ϕ_1至ϕ_6作為代稱，用於簡化反事實模型的描述。表2.1 匯總了六個因果參數的定義及其對應的反事實模型與解釋，定義為平均處理效應的尺度，為後續分析提供基礎。

表 2.1 定義雙有序中介因子之六個因果參數ϕ_1~ϕ_6

因果參數	反事實模型	反事實模型解釋
$\phi_1(a,m_1,m_2)$	$E[Y(a,m_1,m_2)]$	當暴露因子$A=a$、中介因子$M_1=m_1$、$M_2=m_2$時，結果變量的反事實模型。
$\phi_2(a,m_1,e_2,m_1')$	$E\left[Y\left(a,m_1,M_2(e_2,m_1')\right)\right]$	當暴露因子$A=a$、中介因子 $M_1=m_1$ 、 $M_2=M_2(e_1,m_1')$時，結果變量的反事實模型。
$\phi_3(a,m_1,e_2,e_1)$	$E\left[Y\left(a,m_1,M_2\left(e_2,M_1(e_1)\right)\right)\right]$	當暴露因子$A=a$、中介因子$M_1=m_1$、$M_2=M_2\left(e_1,M_1(e_1)\right)$時，結果變量的反事實模型。
$\phi_4(a,e_1,m_2)$	$E[Y(a,M_1(e_1),m_2)]$	當暴露因子$A=a$、中介因子$M_1=M_1(e_1)$、$M_2=m_2$時，結果變量的反事實模型。
$\phi_5(a,e_1,e_2,m_1)$	$E\left[Y\left(a,M_1(e_1),M_2(e_2,m_1)\right)\right]$	當暴露因子$A=a$、中介因子 $M_1=M_1(e_1)$ 、 $M_2=M_2(e_2,m_1)$時，結果變量的反事實模型。
$\phi_6(a,e_1,e_2,e_1)$	$E\left[Y\left(a,M_1(e_1),M_2\left(e_2,M_1(e_1)\right)\right)\right]$	當暴露因子$A=a$、中介因子 $M_1=M_1(e_1)$ 、 $M_2=M_2\left(e_2,M_1(e_1)\right)$時，結果變量的反事實模型。

(2.1.2) 因果參數之效應辨識假設

在真實資料分析中，要證明暴露因子、中介因子和結果變量間的因果性，往往會考慮干擾因子對於整體效應的影響。不過在此為了便於討論，我們在不考慮基線干擾因子(baseline confounding)的條件下，進行ϕ_1至ϕ_6的辨識。下列

$(A1)$~$(A6)$則分別呈現各個暴露因子A、中介因子M_1、M_2以及結果變量Y之間未測量干擾因子(unmeasured confounding)、時變干擾因子(time－varying confounding)，以下將$A \rightarrow Y$的未測量干擾因子定義為U_{AY}、$A \rightarrow M_1$的未測量干擾因子定義為U_{A1}、$A \rightarrow M_2$的未測量干擾因子定義為U_{A2}、$M_1 \rightarrow M_2$的未測量干擾因子定義為U_{12}、$M_1 \rightarrow Y$的未測量干擾因子定義為U_{1Y}、$M_2 \rightarrow Y$的未測量干擾因子定義為U_{2Y}、$A \rightarrow M_1$之間時變干擾因子定義為L_1、$M_1 \rightarrow M_2$之間時變干擾因子定義為L_2，呈現$(A1)$~$(A6)$不應存在的未測量干擾因子、時變干擾因子的結果。

$(A1)$ $Y(a, m_1, m_2) \perp \{A, M_1, M_2\}$

- $(A1.1)$ $Y(a, m_1, m_2) \perp A$，暴露因子A與結果變量Y之間不應存在未測量干擾因子U_{AY}。
- $(A1.2)$ $Y(a, m_1, m_2) \perp M_1|A$，中介因子$M_1$與結果變量$Y$之間不應存在未測量干擾因子$U_{1Y}$。
- $(A1.3)$ $Y(a, m_1, m_2) \perp M_2|A, M_1$，中介因子$M_2$與結果變量$Y$之間不應存在未測量干擾因子$U_{2Y}$。

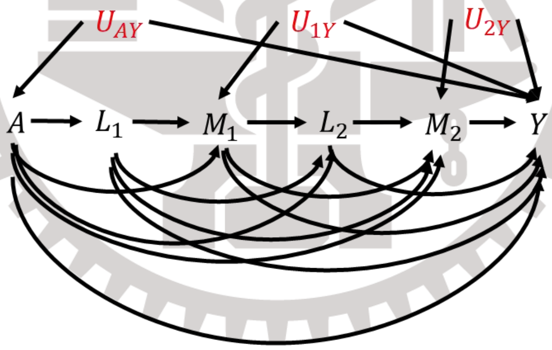


圖 2.1 辨識假設$(A1)$不應存在之干擾因子的因果圖

$(A2)$ $Y(a, m_1, m_2) \perp M_1(e_1)$，中介因子$M_1$與結果變量$Y$之間。不應存在未測量干擾因子$U_{1Y}$、時變干擾因子$L_1$。

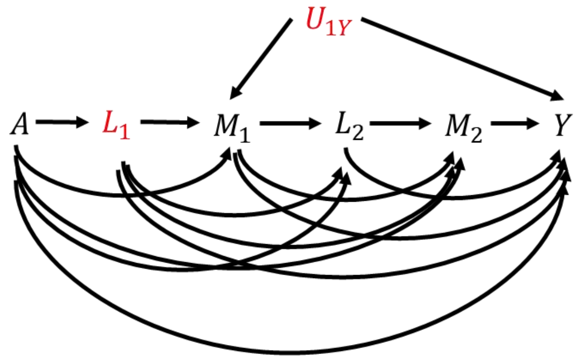


圖 2.2 辨識假設$(A2)$不應存在之干擾因子的因果圖

$(A3)$ $Y(a, m_1, m_2) \perp M_2(e_2, m_1')$，中介因子$M_2$與結果變量$Y$之間。不應存在未測量干擾因子$U_{2Y}$、時變干擾因子$L_1$、$L_2$。

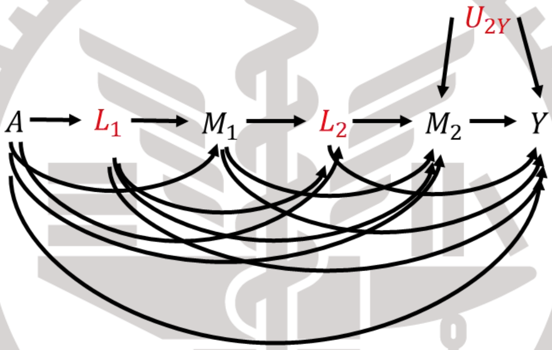


圖 2.3 辨識假設$(A3)$不應存在之干擾因子的因果圖

$(A4)$ $M_1(e_1) \perp A$，暴露因子A與中介因子M_1之間不應存在未測量干擾因子U_{A1}。

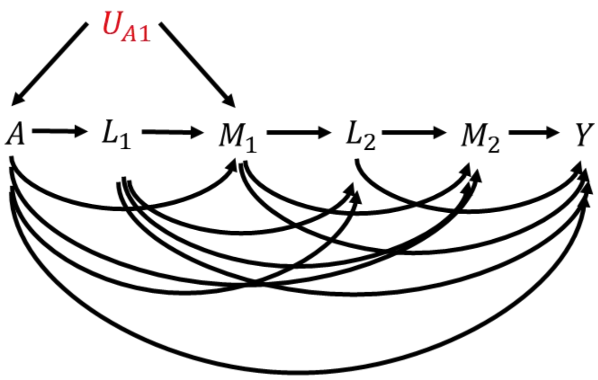


圖 2.4 辨識假設$(A4)$不應存在之干擾因子的因果圖

$(A5)$ $M_2(e_2, m_1) \perp \{A, M_1\}$

- $(A5.1)$ $M_2(e_2, m_1) \perp A$，暴露因子A與中介因子M_2之間不應存在未測量干擾因子U_{A2}。
- $(A5.2)$ $M_2(e_2, m_1) \perp M_1 | A$，中介因子$M_1$與中介因子$M_2$之間不應存在未測量干擾因子$U_{12}$。

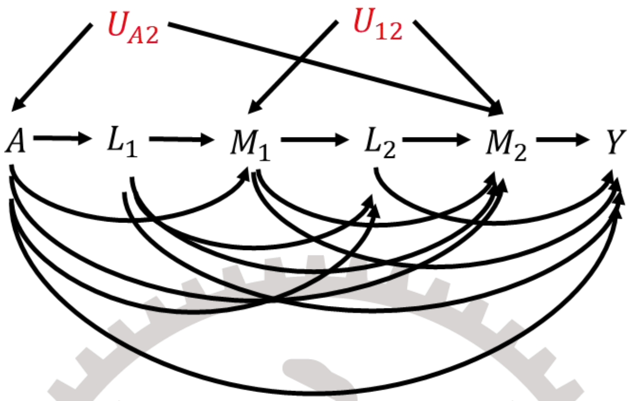


圖 2.5 辨識假設$(A5)$不應存在之干擾因子的因果圖

$(A6)$ $M_2(e_2, m_1) \perp M_1(e_1)$，中介因子$M_1$與中介因子$M_2$之間不應存在未測量干擾因子$U_{12}$、時變干擾因子$L_1$。

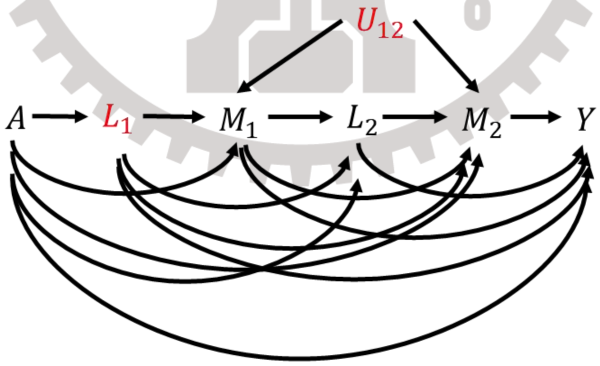


圖 2.6 辨識假設$(A6)$不應存在之干擾因子的因果圖

(2.1.3) 效應辨識結果與 NPSEM

有了因果參數的定義，以及辨識參數所需的假設，我們就可以將使用巢狀反事實定義的因果模型的六個因果參數ϕ_1至ϕ_6，表達成現實中可以計算的期望值與機率值的組合。這些期望值與機率值是指結果變量Y、暴露因子A和中介因子

M_1 、M_2的期望值或條件機率，使得我們能夠透過這些觀察到的數據去計算因果參數，以便於我們接下來計算因果效應。下表則是整理因果參數的辨識結果、辨識假設、各假設所不應存在的未測量干擾因子、無母數結構方程模型(Nonparametric Structual Equation Model, NPSEM)。

表 2.2 因果參數ϕ_1至ϕ_6的辨識結果與 NPSEM 彙整

因果參數		辨識結果
ϕ_1	$E[Y(a, m_1, m_2)]$	$E[Y \mid A = a, M_1 = m_1, M_2 = m_2]$
ϕ_2	$E\big[Y\big(a, m_1, M_2(e_2, m_1')\big)\big]$	$\Sigma_{m_2} E[Y \mid A = a, M_1 = m_1, M_2 = m_2]$ $P(M_2 = m_2 \mid A = e_2, M_1 = m_1')$
ϕ_3	$E\Big[Y\Big(a, m_1, M_2\big(e_2, M_1(e_1)\big)\Big)\Big]$	$\Sigma_{m_1 m_2} E[Y \mid A = a, M_1 = m_1, M_2 = m_2]$ $P(M_2 = m_2 \mid A = e_2, M_1 = m_1) P(M_1 = m_1 \mid A = e_1)$
ϕ_4	$E[Y(a, M_1(e_1), m_2)]$	$\Sigma_{m_1} E[Y \mid A = a, M_1 = m_1, M_2 = m_2]$ $P(M_1 = m_1 \mid A = e_1)$
ϕ_5	$E\big[Y\big(a, M_1(e_1), M_2(e_2, m_1)\big)\big]$	$\Sigma_{m_1 m_2} E[Y \mid A = a, M_1 = m_1, M_2 = m_2]$ $P(M_2 = m_2 \mid A = e_2, M_1 = m_1) P(M_1 = m_1 \mid A = e_1)$
ϕ_6	$E\Big[Y\Big(a, M_1(e_1), M_2\big(e_2, M_1(e_1)\big)\Big)\Big]$	$\Sigma_{m_1 m_2} E[Y \mid A = a, M_1 = m_1, M_2 = m_2]$ $P(M_2 = m_2 \mid A = e_2, M_1 = m_1) P(M_1 = m_1 \mid A = e_1)$

辨識假設

$(A1)$ $Y(a, m_1, m_2) \perp \{A, M_1, M_2\}$

- $(A1.1)$ $Y(a, m_1, m_2) \perp A$
- $(A1.2)$ $Y(a, m_1, m_2) \perp M_1 \mid A$
- $(A1.3)$ $Y(a, m_1, m_2) \perp M_2 \mid A, M_1$

$(A2)$ $Y(a, m_1, m_2) \perp M_1(e_1)$

$(A3)$ $Y(a, m_1, m_2) \perp M_2(e_2, m_1')$

$(A4)$ $M_1(e_1) \perp A$

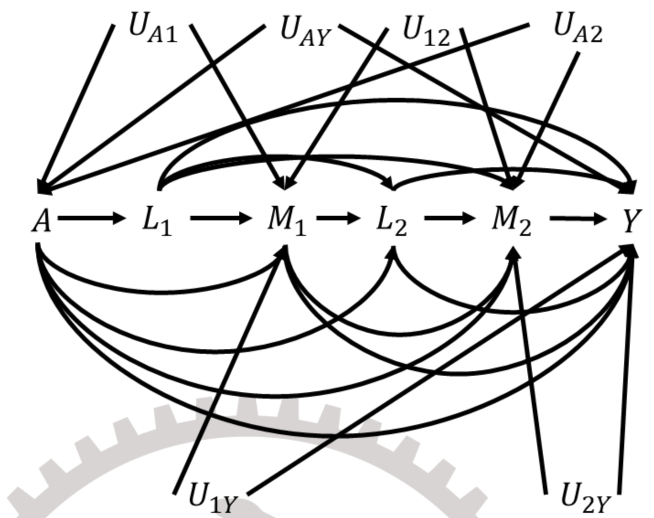
$(A5)$ $M_2(e_2, m_1) \perp \{A, M_1\}$

- $(A5.1)$ $M_2(e_2, m_1) \perp A$
- $(A5.2)$ $M_2(e_2, m_1) \perp M_1 \mid A$

$(A6)$ $M_2(e_2, m_1) \perp M_1(e_1)$

	$(A1.1)$	$(A1.2)$	$(A1.3)$	$(A2)$	$(A3)$	$(A4)$	$(A5.1)$	$(A5.2)$	$(A6)$	不應存在之干擾因子
ϕ_1	■	■	■							U_{AY} 、U_{1Y} 、U_{2Y}
ϕ_2	■	■	■		■		■	■		U_{AY} 、U_{1Y} 、U_{2Y} 、U_{A2} 、U_{12} 、L_1 、L_2
ϕ_3	■	■	■	■	■	■	■	■	■	U_{AY} 、U_{1Y} 、U_{2Y} 、U_{A1} 、U_{A2} 、U_{12} 、L_1 、L_2
ϕ_4	■	■	■	■		■				U_{AY} 、U_{1Y} 、U_{2Y} 、U_{A1} 、L_1
ϕ_5	■	■	■	■	■	■	■	■	■	U_{AY} 、U_{1Y} 、U_{2Y} 、U_{A1} 、U_{A2} 、U_{12} 、L_1 、L_2
ϕ_6	■	■	■	■	■	■	■	■	■	U_{AY} 、U_{1Y} 、U_{2Y} 、U_{A1} 、U_{A2} 、U_{12} 、L_1 、L_2

	$(A1.1)$	$(A1.2)$	$(A1.3)$	$(A2)$	$(A3)$	$(A4)$	$(A5.1)$	$(A5.2)$	$(A6)$	
不應存在的干擾因子	U_{AY}	U_{1Y}	U_{2Y}	U_{1Y}	U_{2Y}	U_{A1}	U_{A2}	U_{12}	U_{12}	
				L_1	L_1				L_1	
					L_2					



2.2 定義二十八路徑拆解法與辨識假設

在此小節中，我們將闡述本研究室(Liao, 2021)所提出二十八路徑拆解法之特定效應路徑，整理成二十八條路徑效應命名為$effect1$至$effect28$，並且說明如何使用前一小節的六個因果參數ϕ_1至ϕ_6定義出這二十八條路徑特定效應之巢狀反事實模型。

(2.2.1) 回顧二十八路徑拆解法最細部拆解效應

為便於理解雙有序中介因子之中介效應與交互作用的機制分析，我們引入其因果圖，如圖 2.7 所示，我們可以發現雙有序中介因子之中介效應與交互作用的機制分析需考量暴露因子A、中介因子M_1、M_2、交互作用項AM_1、AM_2、AM_1M_2、結果變量Y的組合。本研究室(Liao, 2021)則將這些生成因果效應的暴露因子、交互作用項、結果變量的組合一一整理。

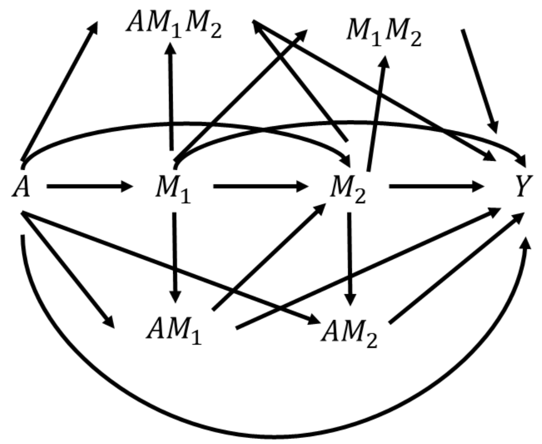


圖 2.7 雙有序中介因子下的因果圖

透過雙有序中介因子下的因果圖，可以直觀理解效應的生成機制，像是總共有七個箭頭指向結果變量Y，表示Y的成因可區分為以下七種生成方式：(1)A生成、(2)M_1生成、(3)M_2生成、(4)AM_1交互生成、(5)AM_2交互生成、(6)AM_1M_2交互生成、(7)M_1M_2交互生成。而中介因子M_1、M_2可能有不通過或是自行產生因果效應的情形，所以除了箭頭指向的效應生成機制外，需要加入「自行生成」、「永不生成」的條件。接著整理有M_2參與的機制且根據M_2的成因區分為：(1)A生成、(2)M_1生成、(3)AM_1交互生成、(4)自行生成、(5)永不生成；M_1的成因區分為：(1)A生成、(2)自行生成、(3)永不生成。透過上述的窮舉及組合的方式，便可以得出二十八路徑拆解法最細部的二十八條路徑特定效應。

舉例來說：暴露因子A直接影響結果變量Y，$A \rightarrow Y$，此效應是Y生成方式中透過「A生成」所產生、中介因子M_1、M_2皆不通過，所以依照生成方式皆屬於「永不生成」，我們將其命名成$effect1$；暴露因子A通過中介因子M_1影響結果變量Y，$A \rightarrow M_1 \rightarrow Y$，此效應是$Y$生成方式中透過「$A$生成」所產生，中介因子$M_1$則需要暴露因子$A$通過，故$M_1$則是「$A$生成」，中介因子$M_2$不需通過則歸類於「永不生成」，我們將其命名成$effect7$。藉此整理出雙有序中介下的路徑特定效應。

由上述的機制分析，我們也發現二十八條路徑特定效應能藉由前一小節的六個因果參數表達，以下則呈現雙有序中介因子的二十八條路徑效應生成機制與因果參數效應定義的結果。

$\phi_1(a, m_1, m_2)$	$E[Y(a, m_1, m_2)]$
$\phi_2(a, m_1, e_2, m_1')$	$E\left[Y\left(a, m_1, M_2(e_2, m_1')\right)\right]$
$\phi_3(a, m_1, e_2, e_1)$	$E\left[Y\left(a, m_1, M_2\left(e_2, M_1(e_1)\right)\right)\right]$
$\phi_4(a, e_1, m_2)$	$E[Y(a, M_1(e_1), m_2)]$

$\phi_5(a, e_1, e_2, m_1)$	$E[Y(a, M_1(e_1), M_2(e_2, m_1))]$
$\phi_6(a, e_1, e_2, e_1)$	$E\left[Y\left(a, M_1(e_1), M_2(e_2, M_1(e_1))\right)\right]$

表 2.3 雙有序中介因子下機制分析窮舉表

效應	M_1生成方式	M_2生成方式	Y生成方式	路徑意義	因果參數效應定義
$effect1$	永不生成	永不生成	A生成	A直接影響Y，沒有通過任何中介因子	$\phi_1(1,0,0) - \phi_1(0,0,0)$
$effect2$	自行生成	永不生成	AM_1交互生成	A通過與M_1的交互作用影響Y，M_1自行生成	$[\phi_4(1,0,0) - \phi_4(0,0,0)]$ $-[\phi_1(1,0,0) - \phi_1(0,0,0)]$
$effect3$	永不生成	自行生成	AM_2交互生成	A通過與M_2的交互作用影響Y，M_2自行生成	$[\phi_2(1,0,0,0) - \phi_2(0,0,0,0)]$ $-[\phi_1(1,0,0) - \phi_1(0,0,0)]$
$effect4$	自行生成	自行生成	AM_1M_2交互生成	A通過與M_1和M_2交互作用(三重交互作用)影響Y，M_1和M_2是自行生成	$[\phi_5(1,0,0,0) - \phi_5(0,0,0,0)]$ $-[\phi_4(1,0,0) - \phi_4(0,0,0)]$ $-[\phi_2(1,0,0,0) - \phi_2(0,0,0,0)]$ $+[\phi_1(1,0,0) - \phi_1(0,0,0)]$
$effect5$	自行生成	M_1生成	AM_1M_2交互生成	A通過與M_1和M_2交互作用(三重交互作用)影響Y，M_1是自行生成，M_2由M_1生成	$[\phi_6(1,0,0,0) - \phi_6(0,0,0,0)]$ $-[\phi_5(1,0,0,0) - \phi_5(0,0,0,0)]$ $-[\phi_3(1,0,0,0) - \phi_3(0,0,0,0)]$ $+[\phi_2(1,0,0,0) - \phi_2(0,0,0,0)]$
$effect6$	自行生成	M_1生成	AM_2交互生成	A通過與M_2的參考交互作用影響Y，M_1自行生成，M_2由M_1生成	$[\phi_3(1,0,0,0) - \phi_3(0,0,0,0)]$ $-[\phi_2(1,0,0,0) - \phi_2(0,0,0,0)]$
$effect7$	A生成	永不生成	M_1生成	A通過與M_1的交互作用影響Y	$\phi_4(0,1,0) - \phi_4(0,0,0)$
$effect8$	永不生成	A生成	M_2生成	A通過與M_2的交互作用影響Y	$\phi_2(0,0,1,0) - \phi_2(0,0,0,0)$
$effect9$	A生成	M_1生成	M_1M_2交互生成	A生成M_1，M_1生成M_2，M_1和M_2的交互作用影響Y	$[\phi_6(0,1,0,1) - \phi_6(0,0,0,0)]$ $-[\phi_5(0,1,0,0) - \phi_5(0,0,0,0)]$ $-[\phi_3(0,0,0,1) - \phi_3(0,0,0,0)]$
$effect10$	A生成	M_1生成	M_2生成	A生成M_1，M_1生成M_2，M_2再影響Y	$\phi_3(0,0,0,1) - \phi_3(0,0,0,0)$
$effect11$	A生成	自行生成	M_1M_2交互生成	A生成M_1，M_2自行生成，M_1和M_2的交互作用影響Y	$[\phi_5(0,1,0,0) - \phi_5(0,0,0,0)]$ $-[\phi_4(0,1,0) - \phi_4(0,0,0)]$
$effect12$	自行生成	AM_1交互生成	M_2生成	M_1是自行生成，A和M_1交互作用影響M_2，M_2再影響Y	$[\phi_3(0,0,1,0) - \phi_3(0,0,0,0)]$ $-[\phi_2(0,0,1,0) - \phi_2(0,0,0,0)]$
$effect13$	A生成	AM_1交互生成	M_2生成	A生成M_1，A和M_1交互作用影響M_2，M_2再影響Y	$[\phi_3(0,0,1,1) - \phi_3(0,0,0,1)]$ $-[\phi_3(0,0,1,0) - \phi_3(0,0,0,0)]$
$effect14$	A生成	A生成	M_1M_2交互生成	A同時生成M_1和M_2，M_1和M_2的交互作用生成Y	$[\phi_5(0,1,1,0) - \phi_5(0,0,1,0)]$ $-[\phi_5(0,1,0,0) - \phi_5(0,0,0,0)]$
$effect15$	自行生成	AM_1交互生成	M_1M_2交互生成	M_1是自行生成，A和M_1交互作用影響M_2，M_1和M_2的交互作用影響Y	$[\phi_6(0,0,1,0) - \phi_6(0,0,0,0)]$ $-[\phi_5(0,0,1,0) - \phi_5(0,0,0,0)]$

					$-[\phi_3(0,0,1,0)-\phi_3(0,0,0,0)]$ $+[\phi_2(0,0,1,0)-\phi_2(0,0,0,0)]$
$effect16$	自行生成	A生成	M_1M_2交互生成	M_1是自行生成，A生成M_2，M_1和M_2的交互作用影響Y	$[\phi_5(0,0,1,0)-\phi_5(0,0,0,0)]$ $-[\phi_2(0,0,1,0)-\phi_2(0,0,0,0)]$
$effect17$	A生成	AM_1交互生成	M_1M_2交互生成	A生成M_1，A和M_1交互作用影響M_2，M_1和M_2的交互作用影響Y	$\left\{\begin{matrix}[\phi_6(0,1,1,1)-\phi_6(0,1,0,1)]\\-[\phi_5(0,1,1,0)-\phi_5(0,0,1,0)]\\-[\phi_3(0,0,1,1)-\phi_3(0,0,0,1)]\end{matrix}\right\}$ $-\left\{\begin{matrix}[\phi_6(0,0,1,0)-\phi_6(0,0,0,0)]\\-[\phi_5(0,1,0,0)-\phi_5(0,0,0,0)]\\-[\phi_3(0,0,1,0)-\phi_3(0,0,0,0)]\end{matrix}\right\}$
$effect18$	A生成	永不生成	AM_1交互生成	A和M_1的交互作用影響Y(即A生成M_1，A和M_1交互作用影響Y)	$[\phi_4(1,1,0)-\phi_4(0,1,0)]$ $-[\phi_4(1,0,0)-\phi_4(0,0,0)]$
$effect19$	永不生成	A生成	AM_2交互生成	A和M_2的交互作用影響Y(即A生成M_2，A和M_2交互作用影響Y)	$[\phi_2(1,0,1,0)-\phi_2(1,0,0,0)]$ $-[\phi_2(0,0,1,0)-\phi_2(0,0,0,0)]$
$effect20$	A生成	M_1生成	AM_2交互生成	A生成M_1，M_1生成M_2，A和M_2交互作用影響Y	$[\phi_3(1,0,0,1)-\phi_3(1,0,0,0)]$ $-[\phi_3(0,0,0,1)-\phi_3(0,0,0,0)]$
$effect21$	A生成	自行生成	AM_1M_2交互生成	A生成M_1， M_2自行生成，A和M_1和M_2三重交互作用影響Y	$[\phi_5(1,1,0,0)-\phi_5(1,0,0,0)]$ $-[\phi_5(0,1,0,0)-\phi_5(0,0,0,0)]$ $-[\phi_4(1,1,0)-\phi_4(0,1,0)]$ $+[\phi_4(1,0,0)-\phi_4(0,0,0)]$
$effect22$	A生成	M_1生成	AM_1M_2交互生成	A生成M_1，M_1生成M_2，A和M_1和M_2三重交互作用影響Y	$\left\{\begin{matrix}[\phi_6(1,1,0,1)-\phi_6(1,0,0,0)]\\-[\phi_5(1,1,0,0)-\phi_5(1,0,0,0)]\\-[\phi_3(1,0,0,1)-\phi_3(1,0,0,0)]\end{matrix}\right\}$ $-\left\{\begin{matrix}[\phi_6(0,1,0,1)-\phi_6(0,0,0,0)]\\-[\phi_5(0,1,0,0)-\phi_5(0,0,0,0)]\\-[\phi_3(0,0,0,1)-\phi_3(0,0,0,0)]\end{matrix}\right\}$
$effect23$	A生成	AM_1交互生成	AM_2交互生成	A生成M_1，A和M_1交互作用影響M_2，A和M_2交互作用影響Y	$\{[\phi_3(1,0,1,1)-\phi_3(1,0,1,0)]$ $-[\phi_3(1,0,0,1)-\phi_3(1,0,0,0)]\}$ $-\{[\phi_3(0,0,1,1)-\phi_3(0,0,1,0)]$ $-[\phi_3(0,0,0,1)-\phi_3(0,0,0,0)]\}$
$effect24$	自行生成	AM_1交互生成	AM_2交互生成	M_1是自行生成，A和M_1交互作用影響M_2，A和M_2的交互作用影響Y	$\left\{\begin{matrix}[\phi_3(1,0,1,0)-\phi_3(1,0,0,0)]\\-[\phi_3(0,0,1,0)-\phi_3(0,0,0,0)]\end{matrix}\right\}$ $-\left\{\begin{matrix}[\phi_2(1,0,1,0)-\phi_2(1,0,0,0)]\\-[\phi_2(0,0,1,0)-\phi_2(0,0,0,0)]\end{matrix}\right\}$
$effect25$	A生成	A生成	AM_1M_2交互生成	A同時生成M_1和M_2，A和M_1和M_2三重交互作用影響Y	$\left\{\begin{matrix}[\phi_5(1,1,1,0)-\phi_5(1,0,1,0)]\\-[\phi_5(1,1,0,0)-\phi_5(1,0,0,0)]\end{matrix}\right\}$ $-\left\{\begin{matrix}[\phi_5(0,1,1,0)-\phi_5(0,0,1,0)]\\-[\phi_5(0,1,0,0)-\phi_5(0,0,0,0)]\end{matrix}\right\}$
$effect26$	A生成	AM_1交互生成	AM_1M_2交互生成	A生成M_1，A和M_1交互作用影響M_2，A和M_1和M_2三重交互作用影響Y	$\left\{\begin{matrix}[\phi_6(1,1,1,1)-\phi_6(1,1,0,1)]\\-[\phi_5(1,1,1,0)-\phi_5(1,1,0,0)]\\-[\phi_6(1,0,1,0)-\phi_6(1,0,0,0)]\\+[\phi_5(1,0,1,0)-\phi_5(1,0,0,0)]\\-[\phi_3(1,0,1,1)-\phi_3(1,0,1,0)]\\+[\phi_3(1,0,0,1)-\phi_3(1,0,0,0)]\end{matrix}\right\}$

					$-\left\{\begin{matrix}[\phi_6(0,1,1,1)-\phi_6(0,1,0,1)]\\-[\phi_5(0,1,1,0)-\phi_5(0,1,0,0)]\\-[\phi_6(0,0,1,0)-\phi_6(0,0,0,0)]\\+[\phi_5(0,0,1,0)-\phi_5(0,0,0,0)]\\-[\phi_3(0,0,1,1)-\phi_3(0,0,1,0)]\\+[\phi_3(0,0,0,1)-\phi_3(0,0,0,0)]\end{matrix}\right\}$
$effect27$	自行生成	AM_1交互生成	AM_1M_2交互生成	M_1是自行生成，A和M_1交互作用影響M_2，A和M_1和M_2三重交互作用影響Y	$\left\{\begin{matrix}[\phi_6(1,0,1,0)-\phi_6(1,0,0,0)]\\-[\phi_5(1,0,1,0)-\phi_5(1,0,0,0)]\\-[\phi_3(1,0,1,0)-\phi_3(1,0,0,0)]\\+[\phi_2(1,0,1,0)-\phi_2(1,0,0,0)]\end{matrix}\right\}$ $-\left\{\begin{matrix}[\phi_6(0,0,1,0)-\phi_6(0,0,0,0)]\\-[\phi_5(0,0,1,0)-\phi_5(0,0,0,0)]\\-[\phi_3(0,0,1,0)-\phi_3(0,0,0,0)]\\+[\phi_2(0,0,1,0)-\phi_2(0,0,0,0)]\end{matrix}\right\}$
$effect28$	自行生成	A生成	AM_1M_2交互生成	M_1是自行生成，A影響M_2，A和M_1和M_2三重交互作用影響Y	$\{[\phi_5(1,0,1,0)-\phi_5(0,0,1,0)]$ $-[\phi_2(1,0,1,0)-\phi_2(1,0,0,0)]\}$ $-\{[\phi_5(1,0,0,0)-\phi_5(0,0,0,0)]$ $-[\phi_2(0,0,1,0)-\phi_2(0,0,0,0)]\}$

表 2.3 所呈現的效應定義主要是以巢狀反事實的框架進行表達，而我們也可以將二十八條路徑效應進行最直觀的呈現，表示成$Y(a,m_1,m_2)$、$M_1(e_1)$、$M_2(e_2,m_1)$的非巢狀反事實模型，便於我們解釋效應的作用機制。不過，使用非巢狀反事實模型的定義，僅能在暴露因子A、中介因子M_1、M_2為二元變數的條件下，雖然較為簡化，但是能協助我們判讀更直覺的判讀交互作用，舉例來說：我們可以由上述的表格得知，$effect2$路徑的意義是「A通過與M_1的交互作用影響Y，M_1自行生成」，因果參數定義即為：

$$E[Y(0,M_1(1),0)-Y(0,M_1(0),0)]-E[Y(1,0,0)-Y(0,0,0)]$$
$$\equiv[\phi_4(0,1,0)-\phi_4(0,0,0)]-[\phi_1(1,0,0)-\phi_1(0,0,0)]$$

可再進一步拆解成以下的效應定義：

$$E\{[Y(1,1,0)-Y(1,0,0)-Y(0,1,0)+Y(0,0,0)]M_1(0)\}$$

當中的$[Y(1,1,0)-Y(1,0,0)-Y(0,1,0)+Y(0,0,0)]$可以較為直觀的解讀成暴露因子$A$、中介因子$M_1$對$Y$交互作用，$M_1(0)$則表示中介因子$M_1$自行生成的效應。下表 2.4 則整理二十八條路徑特定效應以$Y(a,m_1,m_2)$、$M_1(e_1)$、$M_2(e_2,m_1)$的非巢狀反事實模型表達之定義，而關於此二十八條路徑特定效應，從因果參數ϕ_1至ϕ_6推導成$Y(a,m_1,m_2)$、$M_1(e_1)$、$M_2(e_2,m_1)$的反事實模型之效應定義的細節。

表 2.4 雙有序中介因子$effect1$~$effect28$效應直觀之反事實模型定義

效應	$Definition$
$effect1$	$E[Y(1,0,0)-Y(0,0,0)]$
$effect2$	$E\{[Y(1,1,0)-Y(1,0,0)-Y(0,1,0)+Y(0,0,0)]M_1(0)\}$
$effect3$	$E\{[Y(1,0,1)-Y(1,0,0)-Y(0,0,1)+Y(0,0,0)]M_2(0,0)\}$
$effect4$	$E\{[Y(1,1,1)-Y(1,1,0)-Y(1,0,1)+Y(1,0,0)-Y(0,1,1)+Y(0,1,0)+Y(0,0,1)-Y(0,0,0)]M_1(0)M_2(0,0)\}$
$effect5$	$E\{[Y(1,1,1)-Y(1,1,0)-Y(1,0,1)+Y(1,0,0)-Y(0,1,1)+Y(0,1,0)+Y(0,0,1)-Y(0,0,0)]M_1(0)[M_2(0,1)-M_2(0,0)]\}$
$effect6$	$E\{[Y(1,0,1)-Y(1,0,0)-Y(0,0,1)+Y(0,0,0)]M_1(0)[M_2(0,1)-M_2(0,0)]\}$
$effect7$	$E\{[Y(0,1,0)-Y(0,0,0)][M_1(1)-M_1(0)]\}$
$effect8$	$E\{[Y(0,0,1)-Y(0,0,0)][M_2(1,0)-M_2(0,0)]\}$
$effect9$	$E\{[Y(0,1,1)-Y(0,1,0)-Y(0,0,1)+Y(0,0,0)][M_1(1)-M_1(0)][M_2(0,1)-M_2(0,0)]\}$
$effect10$	$E\{[Y(0,0,1)-Y(0,0,0)][M_1(1)-M_1(0)][M_2(0,1)-M_2(0,0)]\}$
$effect11$	$E\{[Y(0,1,1)-Y(0,1,0)-Y(0,0,1)+Y(0,0,0)][M_1(1)-M_1(0)][M_2(0,0)]\}$
$effect12$	$E\{[Y(0,0,1)-Y(0,0,0)][M_1(0)][M_2(1,1)-M_2(1,0)-M_2(0,1)+M_2(0,0)]\}$
$effect13$	$E\{[Y(0,0,1)-Y(0,0,0)][M_1(1)-M_1(0)][M_2(1,1)-M_2(1,0)-M_2(0,1)+M_2(0,0)]\}$
$effect14$	$E\{[Y(0,1,1)-Y(0,1,0)-Y(0,0,1)+Y(0,0,0)][M_1(1)-M_1(0)][M_2(1,0)-M_2(0,0)]\}$
$effect15$	$E\{[Y(0,1,1)-Y(0,1,0)-Y(0,0,1)+Y(0,0,0)][M_1(0)][M_2(1,1)-M_2(1,0)-M_2(0,1)+M_2(0,0)]\}$
$effect16$	$E\{[Y(0,1,1)-Y(0,1,0)-Y(0,0,1)+Y(0,0,0)][M_1(0)][M_2(1,0)-M_2(0,0)]\}$

$effect17$	$E\{[Y(0,1,1)-Y(0,1,0)-Y(0,0,1)+Y(0,0,0)][M_1(1)-M_1(0)][M_2(1,1)-M_2(1,0)-M_2(0,1)+M_2(0,0)]\}$
$effect18$	$E\{[Y(1,1,0)-Y(1,0,0)-Y(0,1,0)+Y(0,0,0)][M_1(1)-M_1(0)]\}$
$effect19$	$E\{[Y(1,0,1)-Y(1,0,0)-Y(0,0,1)+Y(0,0,0)][M_2(1,0)-M_2(0,0)]\}$
$effect20$	$E\{[Y(1,0,1)-Y(1,0,0)-Y(0,0,1)+Y(0,0,0)][M_1(1)-M_1(0)][M_2(0,1)-M_2(0,0)]\}$
$effect21$	$E\{[Y(1,1,1)-Y(1,1,0)-Y(1,0,1)+Y(1,0,0)-Y(0,1,1)+Y(0,1,0)+Y(0,0,1)-Y(0,0,0)][M_1(1)-M_1(0)][M_2(0,0)]\}$
$effect22$	$E\{[Y(1,1,1)-Y(1,1,0)-Y(1,0,1)+Y(1,0,0)-Y(0,1,1)+Y(0,1,0)+Y(0,0,1)-Y(0,0,0)][M_1(1)-M_1(0)][M_2(0,1)-M_2(0,0)]\}$
$effect23$	$E\{[Y(1,0,1)-Y(1,0,0)-Y(0,0,1)+Y(0,0,0)][M_1(1)-M_1(0)][M_2(1,1)-M_2(1,0)-M_2(0,1)+M_2(0,0)]\}$
$effect24$	$E\{[Y(1,0,1)-Y(1,0,0)-Y(0,0,1)+Y(0,0,0)][M_1(0)][M_2(1,1)-M_2(1,0)-M_2(0,1)+M_2(0,0)]\}$
$effect25$	$E\{[Y(1,1,1)-Y(1,1,0)-Y(1,0,1)+Y(1,0,0)-Y(0,1,1)+Y(0,1,0)+Y(0,0,1)-Y(0,0,0)][M_1(1)-M_1(0)][M_2(1,0)-M_2(0,0)]\}$
$effect26$	$E\{[Y(1,1,1)-Y(1,1,0)-Y(1,0,1)+Y(1,0,0)-Y(0,1,1)+Y(0,1,0)+Y(0,0,1)-Y(0,0,0)][M_1(1)-M_1(0)][M_2(1,1)-M_2(1,0)-M_2(0,1)+M_2(0,0)]\}$
$effect27$	$E\{[Y(1,1,1)-Y(1,1,0)-Y(1,0,1)+Y(1,0,0)-Y(0,1,1)+Y(0,1,0)+Y(0,0,1)-Y(0,0,0)][M_1(0)][M_2(1,1)-M_2(1,0)-M_2(0,1)+M_2(0,0)]\}$
$effect28$	$E\{[Y(1,1,1)-Y(1,1,0)-Y(1,0,1)+Y(1,0,0)-Y(0,1,1)+Y(0,1,0)+Y(0,0,1)-Y(0,0,0)][M_1(0)][M_2(1,0)-M_2(0,0)]\}$

以下針對上述定義之數學式，介紹其所代表的機制：

1. $Y(1,0,0)-Y(0,0,0)$為A對於Y的直接影響，$A\rightarrow Y$

2. $Y(0,1,0)-Y(0,0,0)$為M_1對於Y的直接影響，$M_1\rightarrow Y$

3. $Y(0,0,1)-Y(0,0,0)$為M_2對於Y的直接影響，$M_2 \rightarrow Y$

4. $Y(1,1,0)-Y(1,0,0)-Y(0,1,0)+Y(0,0,0)$為$AM_1$對於$Y$的直接影響，也就是$A$、$M_1$對$Y$交互作用，$AM_1 \rightarrow Y$

5. $Y(1,0,1)-Y(1,0,0)-Y(0,0,1)+Y(0,0,0)$為$AM_2$對於$Y$的直接影響，也就是$A$、$M_2$對$Y$交互作用，$AM_2 \rightarrow Y$

6. $Y(0,1,1)-Y(0,1,0)-Y(0,0,1)+Y(0,0,0)$為$M_1M_2$對於$Y$的直接影響，也就是$M_1$、$M_2$對$Y$交互作用$M_1M_2 \rightarrow Y$

7. $Y(1,1,1)-Y(1,1,0)-Y(1,0,1)+Y(1,0,0)-Y(0,1,1)+Y(0,1,0)+Y(0,0,1)-Y(0,0,0)$為$AM_1M_2$對於$Y$的直接影響，也就是$A$、$M_1$、$M_2$對$Y$交互作用，$AM_1M_2 \rightarrow Y$

8. $M_1(1)-M_1(0)$為A對於M_1的直接影響，$A \rightarrow M_1$

9. $M_2(1,0)-M_2(0,0)$為A對於M_2的直接影響，$A \rightarrow M_2$

10. $M_2(0,1)-M_2(0,0)$為M_1對於M_2的直接影響，$M_1 \rightarrow M_2$

11. $M_2(1,1)-M_2(1,0)-M_2(0,1)+M_2(0,0)$為$AM_1$對於$M_2$的直接影響，也就是$A$、$M_1$對$M_2$交互作用，$AM_1 \rightarrow M_2$

對於表 2.4 所整理的數學式定義，可以協助我們將因果參數ϕ_1至ϕ_6表示的二十八條路徑效應，重新表示為以三個反事實模型$Y(a,m_1,m_2)$、$M_1(e_1)$、$M_2(e_2,m_1')$呈現的定義，更為直觀的方式解釋此二十八條效應的機制，並且便於我們引進因果圖的架構。同樣以$effect2$的定義舉例如下：

$$E\{[Y(1,1,0)-Y(1,0,0)-Y(0,1,0)+Y(0,0,0)]M_1(0)\}$$

$Y(1,1,0)-Y(1,0,0)-Y(0,1,0)+Y(0,0,0)$意即$A$、$M_1$對$Y$交互作用，以因果圖表示為$AM_1 \rightarrow Y$，而$effect2$為了形成$AM_1$的交互作用項，也同時會有$A \rightarrow AM_1$、$M_1 \rightarrow AM_1$，其中$M_1(0)$是自行生成的，所以$A$不會有效應指向$M_1$，由此可知構成$effect2$因果圖的架構為$A \rightarrow AM_1 \rightarrow Y$且$M_1 \rightarrow AM_1 \rightarrow Y$，並且$A$和$M_1$之間沒有效應，最後呈現因果圖的結果於圖 2.8。最後我們將二十八條路徑特定效應的因果圖整理至表 2.5

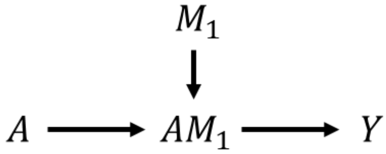
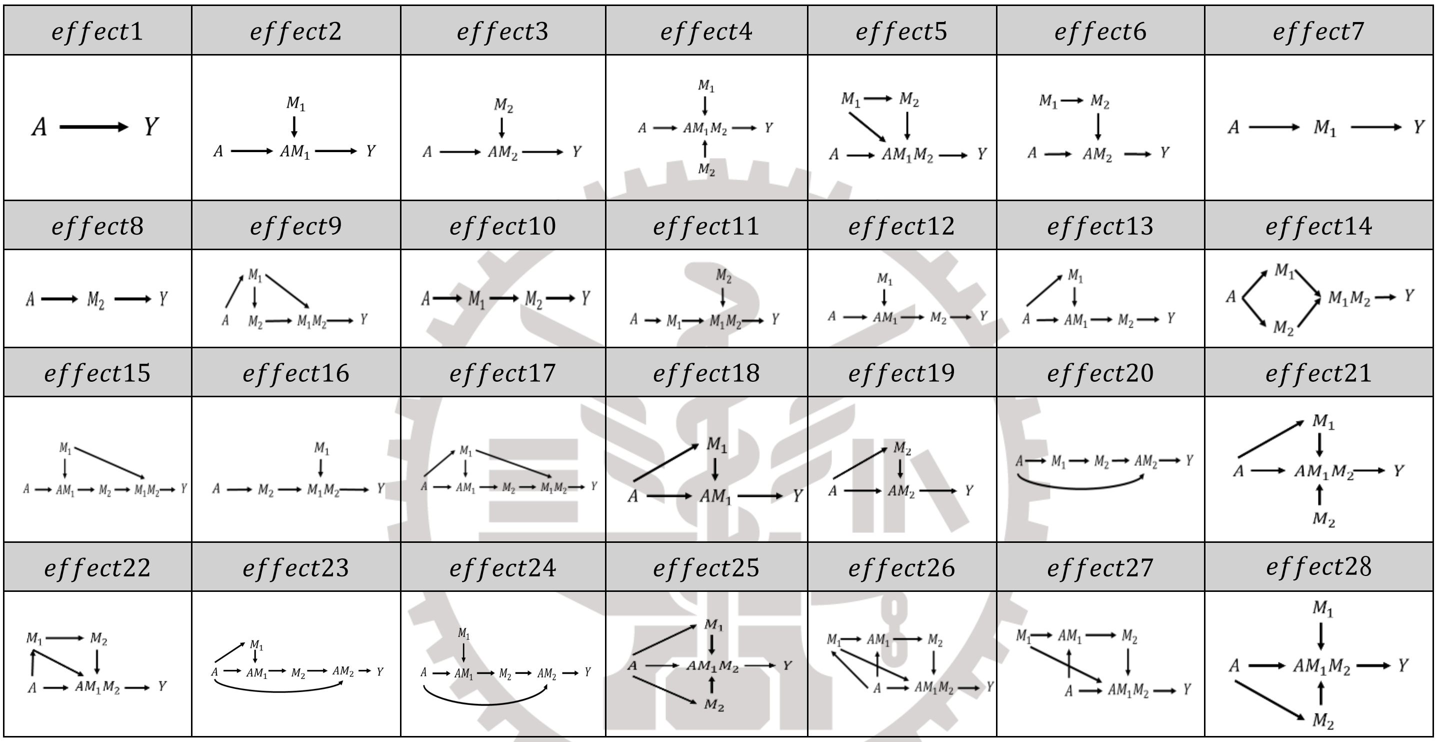


圖 2.8 以$effect2$舉例並整理其因果圖的結果

表 2.5 二十八條路徑特定效應的因果圖

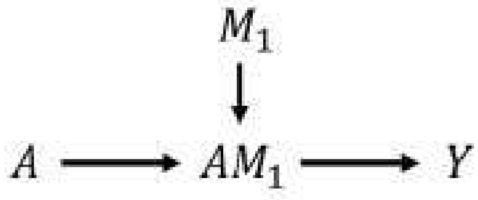


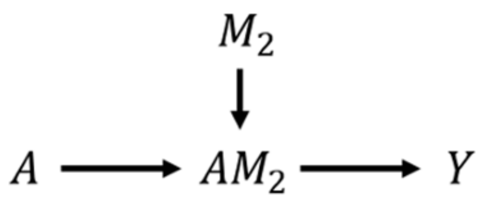
(2.2.2) 二十八路徑拆解法的反事實模型定義、因果圖、辨識假設彙整

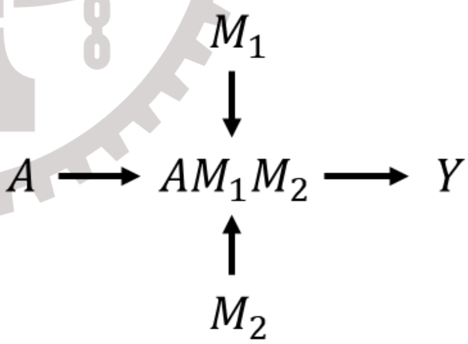
前一小節，我們回顧了雙有序中介因子下，二十八路徑 拆解法中二十八條路徑特定效應的定義及其生成機制。這些二十八條效應皆可以透過因果參數ϕ_1至ϕ_6表示，而每個因果參數都有適當的辨識假設，協助我們將二十八個效應辨識為現實中可以計算的期望值與機率值的組合。此外，因果參數的定義還可以進一步呈現為反事實模型$Y(a, m_1, m_2)$、$M_1(e_1)$、$M_2(e_2, m_1')$的形式，這不僅提供了更為直觀的因果機制理解，還能透過因果圖更清晰地呈現這二十八個效應的機制。為了簡化因果參數ϕ_1至ϕ_6的表示，皆以Δ表示兩個因果參數的相減，例如：$\phi_4(0,1,0) - \phi_4(0,0,0)$簡化為$\Delta_4(010 - 000)$，以下表 2.6 則是二十八條路徑效應的反事實模型定義、因果圖、辨識假設之彙整。

表 2.6 二十八條路徑效應的反事實模型定義、因果圖、辨識假設之彙整

<table>
<tr><th colspan="2">$effect1$</th></tr>
<tr><td colspan="2" align="center">$Definition$</td></tr>
<tr><td colspan="2">$E[Y(1,0,0) - Y(0,0,0)]$
$= [\phi_1(1,0,0) - \phi_1(0,0,0)]$
$= \Delta_1(100 - 000)$</td></tr>
<tr><td colspan="2" align="center">$Identification$</td></tr>
<tr><td colspan="2">$E[Y(a, m_1, m_2)]$
$= E[Y \mid A = a, M_1 = m_1, M_2 = m_2]$</td></tr>
<tr><td align="center">$Required\ Assumptions$</td><td align="center">DAG</td></tr>
<tr><td>$(A1)\ Y(a, m_1, m_2) \perp \{A, M_1, M_2\}$
• $(A1.1)\ Y(a, m_1, m_2) \perp A$
• $(A1.2)\ Y(a, m_1, m_2) \perp M_1 \mid A$
• $(A1.3)\ Y(a, m_1, m_2) \perp M_2 \mid A, M_1$</td><td>
</td></tr>
</table>

<table>
<tr><th colspan="2">$effect2$</th></tr>
<tr><td colspan="2">$Definition$</td></tr>
<tr><td colspan="2">$E\{[Y(1,1,0)-Y(1,0,0)-Y(0,1,0)+Y(0,0,0)]M_1(0)\}$
$=E[Y(1,M_1(0),0)-Y(0,M_1(0),0)]-E[Y(1,0,0)-Y(0,0,0)]$
$=[\phi_4(1,0,0)-\phi_4(0,0,0)]-[\phi_1(1,0,0)-\phi_1(0,0,0)]$
$=\Delta_4(100-000)-\Delta_1(100-000)$</td></tr>
<tr><td colspan="2">$Identification$</td></tr>
<tr><td colspan="2">$E[Y(a,M_1(e_1),m_2)]$
$=\Sigma_{m_1}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_1=m_1|A=e_1)$
$E[Y(a,m_1,m_2)]$
$=E[Y|A=a,M_1=m_1,M_2=m_2]$</td></tr>
<tr><td>$Required\ Assumptions$</td><td>DAG</td></tr>
<tr><td>$(A1)\ Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$
• $(A1.1)\ Y(a,m_1,m_2)\perp A$
• $(A1.2)\ Y(a,m_1,m_2)\perp M_1|A$
• $(A1.3)\ Y(a,m_1,m_2)\perp M_2|A,M_1$
$(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$
$(A4)\ M_1(e_1)\perp A$</td><td>
</td></tr>
</table>

<table>
<tr><th colspan="2">$effect3$</th></tr>
<tr><td colspan="2" align="center">$Definition$</td></tr>
<tr><td colspan="2">$E\{[Y(1,0,1)-Y(1,0,0)-Y(0,0,1)+Y(0,0,0)]M_2(0,0)\}$
$=E[Y(1,0,M_2(0,0))-Y(0,0,M_2(0,0))]-E[Y(1,0,0)-Y(0,0,0)]$
$=[\phi_2(1,0,0,0)-\phi_2(0,0,0,0)]-[\phi_1(1,0,0)-\phi_1(0,0,0)]$
$=\Delta_2(1000-0000)-\Delta_1(100-000)$</td></tr>
<tr><td colspan="2" align="center">$Identification$</td></tr>
<tr><td colspan="2">$E\left[Y\left(a,m_1,M_2(e_2,m_1')\right)\right]$
$=\Sigma_{m_2}E[Y|A=a,M_1=m_1,M_2=m_2]\ P(M_2=m_2|A=e_2,M_1=m_1')$
$E[Y(a,m_1,m_2)]$
$=E[Y|A=a,M_1=m_1,M_2=m_2]$</td></tr>
<tr><td align="center">$Required\ Assumptions$</td><td align="center">DAG</td></tr>
<tr><td>$(A1)\ Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$
• $(A1.1)\ Y(a,m_1,m_2)\perp A$
• $(A1.2)\ Y(a,m_1,m_2)\perp M_1|A$
• $(A1.3)\ Y(a,m_1,m_2)\perp M_2|A,M_1$
$(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$
$(A5)\ M_2(e_2,m_1)\perp\{A,M_1\}$
• $(A5.1)\ M_2(e_2,m_1)\perp A$
• $(A5.2)\ M_2(e_2,m_1)\perp M_1|A$</td><td>
</td></tr>
</table>

<table>
<tr><th colspan="2">$effect4$</th></tr>
<tr><td colspan="2" align="center">$Definition$</td></tr>
<tr><td colspan="2">$E\{[Y(1,1,1)-Y(1,1,0)-Y(1,0,1)+Y(1,0,0)-Y(0,1,1)+Y(0,1,0)+Y(0,0,1)$
$-Y(0,0,0)]M_1(0)M_2(0,0)\}$
$=E\big[Y\big(1,M_1(0),M_2(0,0)\big)-Y\big(0,M_1(0),M_2(0,0)\big)\big]$
$-E[Y(1,M_1(0),0)-Y(0,M_1(0),0)]$
$-E\big[Y\big(1,0,M_2(0,0)\big)-Y\big(0,0,M_2(0,0)\big)\big]$
$+E[Y(1,0,0)-Y(0,0,0)]$
$=[\phi_5(1,0,0,0)-\phi_5(0,0,0,0)]-[\phi_4(1,0,0)-\phi_4(0,0,0)]$
$-[\phi_2(1,0,0,0)-\phi_2(0,0,0,0)]+[\phi_1(1,0,0)-\phi_1(0,0,0)]$
$=\Delta_5(1000-0000)-\Delta_4(100-000)-\Delta_2(1000-0000)+\Delta_1(100-000)$</td></tr>
<tr><td colspan="2" align="center">$Identification$</td></tr>
<tr><td colspan="2">$E\big[Y\big(a,M_1(e_1),M_2(e_2,m_1)\big)\big]$
$=\Sigma_{m_1m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1)P(M_1=m_1|A=e_1)$
$E[Y(a,M_1(e_1),m_2)]$
$=\Sigma_{m_1}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_1=m_1|A=e_1)$
$E\big[Y\big(a,m_1,M_2(e_2,m_1')\big)\big]$
$=\Sigma_{m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1')$
$E[Y(a,m_1,m_2)]$
$=E[Y|A=a,M_1=m_1,M_2=m_2]$</td></tr>
<tr><td align="center">$Required\ Assumptions$</td><td align="center">DAG</td></tr>
<tr><td>$(A1)\ Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$
• $(A1.1)\ Y(a,m_1,m_2)\perp A$
• $(A1.2)\ Y(a,m_1,m_2)\perp M_1|A$
• $(A1.3)\ Y(a,m_1,m_2)\perp M_2|A,M_1$
$(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$
$(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$
$(A4)\ M_1(e_1)\perp A$
$(A5)\ M_2(e_2,m_1)\perp\{A,M_1\}$
• $(A5.1)\ M_2(e_2,m_1)\perp A$
• $(A5.2)\ M_2(e_2,m_1)\perp M_1|A$
$(A6)\ M_2(e_2,m_1)\perp M_1(e_1)$</td><td>
</td></tr>
</table>

$effect5$

$Definition$

$$E\{[Y(1,1,1) - Y(1,1,0) - Y(1,0,1) + Y(1,0,0) - Y(0,1,1) + Y(0,1,0) + Y(0,0,1) - Y(0,0,0)]M_1(0)[M_2(0,1) - M_2(0,0)]\}$$

$$= E\left[Y\left(1, M_1(0), M_2\left(0, M_1(0)\right)\right) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right] - E\left[Y\left(1, M_1(0), M_2(0,0)\right) - Y\left(0, M_1(0), M_2(0,0)\right)\right] - E\left[Y\left(1,0, M_2\left(0, M_1(0)\right)\right) - Y\left(0,0, M_2\left(0, M_1(0)\right)\right)\right] + E\left[Y\left(1,0, M_2(0,0)\right) - Y\left(0,0, M_2(0,0)\right)\right]$$

$$= [\phi_6(1,0,0,0) - \phi_6(0,0,0,0)] - [\phi_5(1,0,0,0) - \phi_5(0,0,0,0)] - [\phi_3(1,0,0,0) - \phi_3(0,0,0,0)] + [\phi_2(1,0,0,0) - \phi_2(0,0,0,0)]$$

$$= \Delta_6(1000 - 0000) - \Delta_5(1000 - 0000) - \Delta_3(1000 - 0000) + \Delta_2(1000 - 0000)$$

$Identification$

$E\left[Y\left(a, M_1(e_1), M_2\left(e_2, M_1(e_1)\right)\right)\right]$

$= \Sigma_{m_1 m_2} E[Y|A = a, M_1 = m_1, M_2 = m_2]P(M_2 = m_2|A = e_2, M_1 = m_1)P(M_1 = m_1|A = e_1)$

$E\left[Y\left(a, M_1(e_1), M_2(e_2, m_1)\right)\right]$

$= \Sigma_{m_1 m_2} E[Y|A = a, M_1 = m_1, M_2 = m_2]P(M_2 = m_2|A = e_2, M_1 = m_1)P(M_1 = m_1|A = e_1)$

$E\left[Y\left(a, m_1, M_2\left(e_2, M_1(e_1)\right)\right)\right]$

$= \Sigma_{m_1 m_2} E[Y|A = a, M_1 = m_1, M_2 = m_2]P(M_2 = m_2|A = e_2, M_1 = m_1)P(M_1 = m_1|A = e_1)$

$E[Y(a, m_1, m_2)]$

$= E[Y|A = a, M_1 = m_1, M_2 = m_2]$

$Required\ Assumptions$

$(A1)$ $Y(a, m_1, m_2) \perp \{A, M_1, M_2\}$

- $(A1.1)$ $Y(a, m_1, m_2) \perp A$
- $(A1.2)$ $Y(a, m_1, m_2) \perp M_1|A$
- $(A1.3)$ $Y(a, m_1, m_2) \perp M_2|A, M_1$

$(A2)$ $Y(a, m_1, m_2) \perp M_1(e_1)$

$(A3)$ $Y(a, m_1, m_2) \perp M_2(e_2, m_1')$

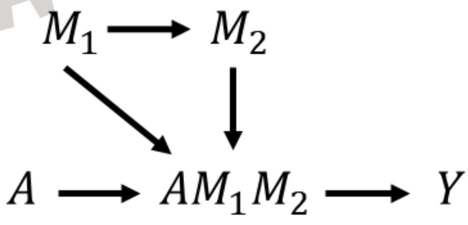
$(A4)$ $M_1(e_1) \perp A$

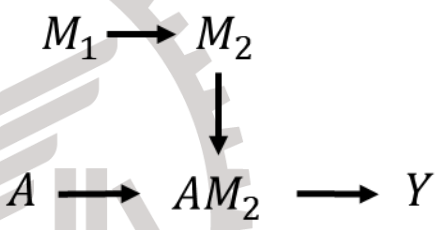
$(A5)$ $M_2(e_2, m_1) \perp \{A, M_1\}$

- $(A5.1)$ $M_2(e_2, m_1) \perp A$
- $(A5.2)$ $M_2(e_2, m_1) \perp M_1|A$

$(A6)$ $M_2(e_2, m_1) \perp M_1(e_1)$

DAG



<table>
<tr><th colspan="2">$effect6$</th></tr>
<tr><td colspan="2">$Definition$</td></tr>
<tr><td colspan="2">$E\{[Y(1,0,1)-Y(1,0,0)-Y(0,0,1)+Y(0,0,0)]M_1(0)[M_2(0,1)-M_2(0,0)]\}$
$= E\left[Y\left(1,0,M_2\left(0,M_1(0)\right)\right)-Y\left(0,0,M_2\left(0,M_2(0)\right)\right)\right]$
$- E\left[Y\left(1,0,M_2(0,0)\right)-Y\left(0,0,M_2(0,0)\right)\right]$
$= [\phi_3(1,0,0,0)-\phi_3(0,0,0,0)]-[\phi_2(1,0,0,0)-\phi_2(0,0,0,0)]$
$= \Delta_3(1000-0000)-\Delta_2(1000-0000)$</td></tr>
<tr><td colspan="2">$Identification$</td></tr>
<tr><td colspan="2">$E\left[Y\left(a,m_1,M_2\left(e_2,M_1(e_1)\right)\right)\right]$
$=\Sigma_{m_1 m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1)P(M_1=m_1|A=e_1)$
$E\left[Y\left(a,m_1,M_2(e_2,m_1')\right)\right]$
$=\Sigma_{m_2}E[Y|A=a,M_1=m_1,M_2=m_2]\ P(M_2=m_2|A=e_2,M_1=m_1')$</td></tr>
<tr><td>$Required\ Assumptions$</td><td>DAG</td></tr>
<tr><td>$(A1)\ Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$
• $(A1.1)\ Y(a,m_1,m_2)\perp A$
• $(A1.2)\ Y(a,m_1,m_2)\perp M_1|A$
• $(A1.3)\ Y(a,m_1,m_2)\perp M_2|A,M_1$
$(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$
$(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$
$(A4)\ M_1(e_1)\perp A$
$(A5)\ M_2(e_2,m_1)\perp\{A,M_1\}$
• $(A5.1)\ M_2(e_2,m_1)\perp A$
• $(A5.2)\ M_2(e_2,m_1)\perp M_1|A$
$(A6)\ M_2(e_2,m_1)\perp M_1(e_1)$</td><td>
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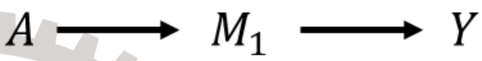
<tr><th colspan="2">$effect7$</th></tr>
<tr><td colspan="2" align="center">$Definition$</td></tr>
<tr><td colspan="2">$E\{[Y(0,1,0)-Y(0,0,0)][M_1(1)-M_1(0)]\}$
$=E[Y(0,M_1(1),0)-Y(0,M_1(0),0)]$
$=\phi_4(0,1,0)-\phi_4(0,0,0)$
$=\Delta_4(010-000)$</td></tr>
<tr><td colspan="2" align="center">$Identification$</td></tr>
<tr><td colspan="2">$E[Y(a,M_1(e_1),m_2)]$
$=\Sigma_{m_1}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_1=m_1|A=e_1)$</td></tr>
<tr><td align="center">$Required\ Assumptions$</td><td align="center">DAG</td></tr>
<tr><td>$(A1)\ Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$
• $(A1.1)\ Y(a,m_1,m_2)\perp A$
• $(A1.2)\ Y(a,m_1,m_2)\perp M_1|A$
• $(A1.3)\ Y(a,m_1,m_2)\perp M_2|A,M_1$
$(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$
$(A4)\ M_1(e_1)\perp A$</td><td>
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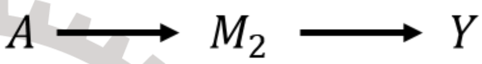
<tr><th colspan="2">$effect8$</th></tr>
<tr><td colspan="2" align="center">$Definition$</td></tr>
<tr><td colspan="2">$E\{[Y(0,0,1)-Y(0,0,0)][M_2(1,0)-M_2(0,0)]\}$
$=E\left[Y\left(0,0,M_2(1,0)\right)-Y\left(0,0,M_2(0,0)\right)\right]$
$=\phi_2(0,0,1,0)-\phi_2(0,0,0,0)$
$=\Delta_2(0010-0000)$</td></tr>
<tr><td colspan="2" align="center">$Identification$</td></tr>
<tr><td colspan="2">$E\left[Y\left(a,m_1,M_2(e_2,m_1')\right)\right]$
$=\Sigma_{m_2}E[Y|A=a,M_1=m_1,M_2=m_2]\ P(M_2=m_2|A=e_2,M_1=m_1')$</td></tr>
<tr><td align="center">$Required\ Assumptions$</td><td align="center">DAG</td></tr>
<tr><td>$(A1)\ Y(a,m_1,m_2)\perp\{A\ ,M_1,M_2\}$
• $(A1.1)\ Y(a,m_1,m_2)\perp A$
• $(A1.2)\ Y(a,m_1,m_2)\perp M_1|A$
• $(A1.3)\ Y(a,m_1,m_2)\perp M_2|A,M_1$
$(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$
$(A5)\ M_2(e_2,m_1)\perp\{A,M_1\}$
• $(A5.1)\ M_2(e_2,m_1)\perp A$
• $(A5.2)\ M_2(e_2,m_1)\perp M_1|A$</td><td>
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<table>
<tr><th colspan="2">$effect9$</th></tr>
<tr><th colspan="2">$Definition$</th></tr>
<tr><td colspan="2">

$$E\{[Y(0,1,1) - Y(0,1,0) - Y(0,0,1) + Y(0,0,0)][M_1(1) - M_1(0)][M_2(0,1) - M_2(0,0)]\}$$

$$= E\left[Y\left(0, M_1(1), M_2\left(0, M_1(1)\right)\right) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$$- E\left[Y\left(0, M_1(1), M_2(0,0)\right) - Y\left(0, M_1(0), M_2(0,0)\right)\right]$$

$$- E\left[Y\left(0,0, M_2\left(0, M_1(1)\right)\right) - Y\left(0,0, M_2\left(0, M_1(0)\right)\right)\right]$$

$$+ E[Y(0, M_1(1), 0) - Y(0, M_1(0), 0)]$$

$$= [\phi_6(0,1,0,1) - \phi_6(0,0,0,0)] - [\phi_5(0,1,0,0) - \phi_5(0,0,0,0)] - [\phi_3(0,0,0,1) - \phi_3(0,0,0,0)]$$

$$= [\Delta_6(0101 - 0000)] - [\Delta_5(0100 - 0000)] - [\Delta_3(0001 - 0000)]$$

</td></tr>
<tr><th colspan="2">$Identification$</th></tr>
<tr><td colspan="2">

$E\left[Y\left(a, M_1(e_1), M_2\left(e_2, M_1(e_1)\right)\right)\right]$

$= \Sigma_{m_1 m_2} E[Y|A = a, M_1 = m_1, M_2 = m_2] P(M_2 = m_2|A = e_2, M_1 = m_1) P(M_1 = m_1|A = e_1)$

$E\left[Y\left(a, M_1(e_1), M_2(e_2, m_1)\right)\right]$

$= \Sigma_{m_1 m_2} E[Y|A = a, M_1 = m_1, M_2 = m_2] P(M_2 = m_2|A = e_2, M_1 = m_1) P(M_1 = m_1|A = e_1)$

$E\left[Y\left(a, m_1, M_2\left(e_2, M_1(e_1)\right)\right)\right]$

$= \Sigma_{m_1 m_2} E[Y|A = a, M_1 = m_1, M_2 = m_2] P(M_2 = m_2|A = e_2, M_1 = m_1) P(M_1 = m_1|A = e_1)$

</td></tr>
<tr><th>$Required\ Assumptions$</th><th>DAG</th></tr>
<tr><td>

$(A1)\ Y(a, m_1, m_2) \perp \{A, M_1, M_2\}$

- $(A1.1)\ Y(a, m_1, m_2) \perp A$
- $(A1.2)\ Y(a, m_1, m_2) \perp M_1|A$
- $(A1.3)\ Y(a, m_1, m_2) \perp M_2|A, M_1$

$(A2)\ Y(a, m_1, m_2) \perp M_1(e_1)$

$(A3)\ Y(a, m_1, m_2) \perp M_2(e_2, m_1')$

$(A4)\ M_1(e_1) \perp A$

$(A5)\ M_2(e_2, m_1) \perp \{A, M_1\}$

- $(A5.1)\ M_2(e_2, m_1) \perp A$
- $(A5.2)\ M_2(e_2, m_1) \perp M_1|A$

$(A6)\ M_2(e_2, m_1) \perp M_1(e_1)$

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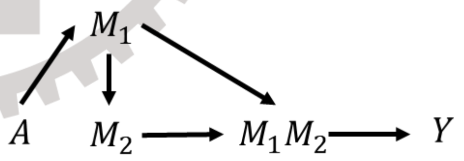

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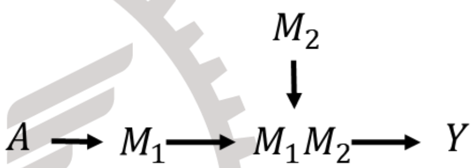
<table>
<tr><th colspan="2">$effect10$</th></tr>
<tr><td colspan="2" align="center">$Definition$</td></tr>
<tr><td colspan="2">$E\{[Y(0,0,1)-Y(0,0,0)][M_1(1)-M_1(0)][M_2(0,1)-M_2(0,0)]\}$
$= E\big[Y\big(0,0,M_2(0,M_1(1))\big)-Y\big(0,0,M_2(0,M_1(0))\big)\big]$
$= \phi_3(0,0,0,1)-\phi_3(0,0,0,0)$
$= \Delta_3(0001-0000)$</td></tr>
<tr><td colspan="2" align="center">$Identification$</td></tr>
<tr><td colspan="2">$E\left[Y\left(a,m_1,M_2(e_2,M_1(e_1))\right)\right]$
$= \Sigma_{m_1 m_2} E[Y \mid A=a, M_1=m_1, M_2=m_2] P(M_2=m_2 \mid A=e_2, M_1=m_1) P(M_1=m_1 \mid A=e_1)$</td></tr>
<tr><td align="center">$Required\ Assumptions$</td><td align="center">DAG</td></tr>
<tr><td>$(A1)\ Y(a,m_1,m_2) \perp \{A, M_1, M_2\}$
• $(A1.1)\ Y(a,m_1,m_2) \perp A$
• $(A1.2)\ Y(a,m_1,m_2) \perp M_1 \mid A$
• $(A1.3)\ Y(a,m_1,m_2) \perp M_2 \mid A, M_1$
$(A2)\ Y(a,m_1,m_2) \perp M_1(e_1)$
$(A3)\ Y(a,m_1,m_2) \perp M_2(e_2,m_1')$
$(A4)\ M_1(e_1) \perp A$
$(A5)\ M_2(e_2,m_1) \perp \{A, M_1\}$
• $(A5.1)\ M_2(e_2,m_1) \perp A$
• $(A5.2)\ M_2(e_2,m_1) \perp M_1 \mid A$
$(A6)\ M_2(e_2,m_1) \perp M_1(e_1)$</td><td>$A \longrightarrow M_1 \longrightarrow M_2 \longrightarrow Y$</td></tr>
</table>

<table>
<tr><th colspan="2">$effect11$</th></tr>
<tr><td colspan="2" align="center">$Definition$</td></tr>
<tr><td colspan="2">$E\{[Y(0,1,1) - Y(0,1,0) - Y(0,0,1) + Y(0,0,0)][M_1(1) - M_1(0)][M_2(0,0)]\}$
$= E\big[Y\big(0, M_1(1), M_2(0,0)\big) - Y\big(0, M_1(0), M_2(0,0)\big)\big]$
$- E[Y(0, M_1(1), 0) - Y(0, M_1(0), 0)]$
$= [\phi_5(0,1,0,0) - \phi_5(0,0,0,0)] - [\phi_4(0,1,0) - \phi_4(0,0,0)]$
$= \Delta_5(0100 - 0000) - \Delta_4(010 - 000)$</td></tr>
<tr><td colspan="2" align="center">$Identification$</td></tr>
<tr><td colspan="2">$E\big[Y\big(a, M_1(e_1), M_2(e_2, m_1)\big)\big]$
$= \Sigma_{m_1 m_2} E[Y|A = a, M_1 = m_1, M_2 = m_2] P(M_2 = m_2|A = e_2, M_1 = m_1) P(M_1 = m_1|A = e_1)$
$E[Y(a, M_1(e_1), m_2)]$
$= \Sigma_{m_1} E[Y|A = a, M_1 = m_1, M_2 = m_2] P(M_1 = m_1|A = e_1)$</td></tr>
<tr><td align="center">$Required\ Assumptions$</td><td align="center">DAG</td></tr>
<tr><td>$(A1)\ Y(a, m_1, m_2) \perp \{A, M_1, M_2\}$
• $(A1.1)\ Y(a, m_1, m_2) \perp A$
• $(A1.2)\ Y(a, m_1, m_2) \perp M_1|A$
• $(A1.3)\ Y(a, m_1, m_2) \perp M_2|A, M_1$
$(A2)\ Y(a, m_1, m_2) \perp M_1(e_1)$
$(A3)\ Y(a, m_1, m_2) \perp M_2(e_2, m_1')$
$(A4)\ M_1(e_1) \perp A$
$(A5)\ M_2(e_2, m_1) \perp \{A, M_1\}$
• $(A5.1)\ M_2(e_2, m_1) \perp A$
• $(A5.2)\ M_2(e_2, m_1) \perp M_1|A$
$(A6)\ M_2(e_2, m_1) \perp M_1(e_1)$</td><td>
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<table>
<tr><th colspan="2">$effect12$</th></tr>
<tr><td colspan="2" align="center">$Definition$</td></tr>
<tr><td colspan="2">

$$E\{[Y(0,0,1)-Y(0,0,0)][M_1(0)][M_2(1,1)-M_2(1,0)-M_2(0,1)+M_2(0,0)]\}$$

$$= E\left[Y\left(0,0,M_2\big(1,M_1(0)\big)\right)-Y\left(0,0,M_2\big(0,M_2(0)\big)\right)\right]$$

$$-E\big[Y\big(0,0,M_2(1,0)\big)-Y\big(0,0,M_2(0,0)\big)\big]$$

$$= [\phi_3(0,0,1,0)-\phi_3(0,0,0,0)]-[\phi_2(0,0,1,0)-\phi_2(0,0,0,0)]$$

$$= \Delta_3(0010-0000)-\Delta_2(0010-0000)$$

</td></tr>
<tr><td colspan="2" align="center">$Identification$</td></tr>
<tr><td colspan="2">

$E\left[Y\left(a,m_1,M_2\big(e_2,M_1(e_1)\big)\right)\right]$

$=\Sigma_{m_1m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1)P(M_1=m_1|A=e_1)$

$E\big[Y\big(a,m_1,M_2(e_2,m_1')\big)\big]$

$=\Sigma_{m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1')$

</td></tr>
<tr><td align="center">$Required\ Assumptions$</td><td align="center">DAG</td></tr>
<tr><td>

$(A1)\ Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$

- $(A1.1)\ Y(a,m_1,m_2)\perp A$
- $(A1.2)\ Y(a,m_1,m_2)\perp M_1|A$
- $(A1.3)\ Y(a,m_1,m_2)\perp M_2|A,M_1$

$(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$

$(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$

$(A4)\ M_1(e_1)\perp A$

$(A5)\ M_2(e_2,m_1)\perp\{A,M_1\}$

- $(A5.1)\ M_2(e_2,m_1)\perp A$
- $(A5.2)\ M_2(e_2,m_1)\perp M_1|A$

$(A6)\ M_2(e_2,m_1)\perp M_1(e_1)$

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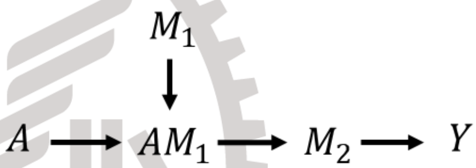

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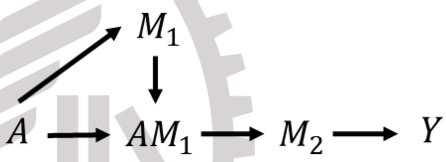
<tr><th colspan="2">$effect13$</th></tr>
<tr><td colspan="2" align="center">$Definition$</td></tr>
<tr><td colspan="2">$E\{[Y(0,0,1)-Y(0,0,0)][M_1(1)-M_1(0)][M_2(1,1)-M_2(1,0)-M_2(0,1)+M_2(0,0)]\}$
$=E\left[Y\left(0,0,M_2\left(1,M_1(1)\right)\right)-Y\left(0,0,M_2\left(0,M_1(1)\right)\right)\right]$
$-E\left[Y\left(0,0,M_2\left(1,M_1(0)\right)\right)-Y\left(0,0,M_2\left(0,M_1(0)\right)\right)\right]$
$=[\phi_3(0,0,1,1)-\phi_3(0,0,0,1)]-[\phi_3(0,0,1,0)-\phi_3(0,0,0,0)]$
$=\Delta_3(0011-0001)-\Delta_3(0010-0000)$</td></tr>
<tr><td colspan="2" align="center">$Identification$</td></tr>
<tr><td colspan="2">$E\left[Y\left(a,m_1,M_2\left(e_2,M_1(e_1)\right)\right)\right]$
$=\Sigma_{m_1m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1)P(M_1=m_1|A=e_1)$</td></tr>
<tr><td align="center">$Required\ Assumptions$</td><td align="center">DAG</td></tr>
<tr><td>$(A1)\ Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$
• $(A1.1)\ Y(a,m_1,m_2)\perp A$
• $(A1.2)\ Y(a,m_1,m_2)\perp M_1|A$
• $(A1.3)\ Y(a,m_1,m_2)\perp M_2|A,M_1$
$(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$
$(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$
$(A4)\ M_1(e_1)\perp A$
$(A5)\ M_2(e_2,m_1)\perp\{A,M_1\}$
• $(A5.1)\ M_2(e_2,m_1)\perp A$
• $(A5.2)\ M_2(e_2,m_1)\perp M_1|A$
$(A6)\ M_2(e_2,m_1)\perp M_1(e_1)$</td><td>
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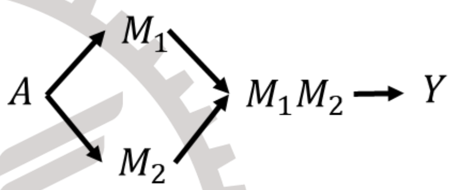
<table>
<tr><th colspan="2">$effect14$</th></tr>
<tr><th colspan="2">$Definition$</th></tr>
<tr><td colspan="2">$E\{[Y(0,1,1) - Y(0,1,0) - Y(0,0,1) + Y(0,0,0)][M_1(1) - M_1(0)][M_2(1,0) - M_2(0,0)]\}$
$= E\big[Y\big(0, M_1(1), M_2(1,0)\big) - Y\big(0, M_1(0), M_2(1,0)\big)\big] - E\big[Y\big(0, M_1(1), M_2(0,0)\big) - Y\big(0, M_1(0), M_2(0,0)\big)\big]$
$= [\phi_5(0,1,1,0) - \phi_5(0,0,1,0)] - [\phi_5(0,1,0,0) - \phi_5(0,0,0,0)]$
$= \Delta_5(0110 - 0010) - \Delta_5(0100 - 0000)$</td></tr>
<tr><th colspan="2">$Identification$</th></tr>
<tr><td colspan="2">$E\big[Y\big(a, M_1(e_1), M_2(e_2, m_1)\big)\big]$
$= \Sigma_{m_1 m_2} E[Y|A = a, M_1 = m_1, M_2 = m_2]P(M_2 = m_2|A = e_2, M_1 = m_1)P(M_1 = m_1|A = e_1)$</td></tr>
<tr><th>$Required\ Assumptions$</th><th>DAG</th></tr>
<tr><td>$(A1)\ Y(a, m_1, m_2) \perp \{A, M_1, M_2\}$
• $(A1.1)\ Y(a, m_1, m_2) \perp A$
• $(A1.2)\ Y(a, m_1, m_2) \perp M_1|A$
• $(A1.3)\ Y(a, m_1, m_2) \perp M_2|A, M_1$
$(A2)\ Y(a, m_1, m_2) \perp M_1(e_1)$
$(A3)\ Y(a, m_1, m_2) \perp M_2(e_2, m_1')$
$(A4)\ M_1(e_1) \perp A$
$(A5)\ M_2(e_2, m_1) \perp \{A, M_1\}$
• $(A5.1)\ M_2(e_2, m_1) \perp A$
• $(A5.2)\ M_2(e_2, m_1) \perp M_1|A$
$(A6)\ M_2(e_2, m_1) \perp M_1(e_1)$</td><td>
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<table>
<tr><th colspan="2">$effect15$</th></tr>
<tr><th colspan="2">$Definition$</th></tr>
<tr><td colspan="2">

$E\{[Y(0,1,1)-Y(0,1,0)-Y(0,0,1)+Y(0,0,0)][M_1(0)][M_2(1,1)-M_2(1,0)$
$\quad -M_2(0,1)+M_2(0,0)]\}$

$=E\left[Y\left(0,M_1(0),M_2\left(1,M_1(0)\right)\right)-Y\left(0,M_1(0),M_2\left(0,M_1(0)\right)\right)\right]$
$\quad -E\left[Y\left(0,M_1(0),M_2(1,0)\right)-Y\left(0,M_1(0),M_2(0,0)\right)\right]$
$\quad -E\left[Y\left(0,0,M_2\left(1,M_1(0)\right)\right)-Y\left(0,0,M_2\left(0,M_1(0)\right)\right)\right]$
$\quad +E\left[Y\left(0,0,M_2(1,0)\right)-Y\left(0,0,M_2(0,0)\right)\right]$

$=[\phi_6(0,0,1,0)-\phi_6(0,0,0,0)]-[\phi_5(0,0,1,0)-\phi_5(0,0,0,0)]$
$\quad -[\phi_3(0,0,1,0)-\phi_3(0,0,0,0)]+[\phi_2(0,0,1,0)-\phi_2(0,0,0,0)]$

$=\Delta_6(0010-0000)-\Delta_5(0010-0000)-\Delta_3(0010-0000)+\Delta_2(0010$
$\quad -0000)$

</td></tr>
<tr><th colspan="2">$Identification$</th></tr>
<tr><td colspan="2">

$E\left[Y\left(a,M_1(e_1),M_2\left(e_2,M_1(e_1)\right)\right)\right]$

$=\Sigma_{m_1m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1)P(M_1=m_1|A=e_1)$

$E\left[Y\left(a,M_1(e_1),M_2(e_2,m_1)\right)\right]$

$=\Sigma_{m_1m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1)P(M_1=m_1|A=e_1)$

$E\left[Y\left(a,m_1,M_2\left(e_2,M_1(e_1)\right)\right)\right]$

$=\Sigma_{m_1m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1)P(M_1=m_1|A=e_1)$

$E\left[Y\left(a,m_1,M_2(e_2,m_1')\right)\right]$

$=\Sigma_{m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1')$

</td></tr>
<tr><th>$Required\ Assumptions$</th><th>DAG</th></tr>
<tr><td>

$(A1)$ $Y(a,m_1,m_2)\perp\{A\ ,M_1,M_2\}$

- $(A1.1)$ $Y(a,m_1,m_2)\perp A$
- $(A1.2)$ $Y(a,m_1,m_2)\perp M_1|A$
- $(A1.3)$ $Y(a,m_1,m_2)\perp M_2|A,M_1$

$(A2)$ $Y(a,m_1,m_2)\perp M_1(e_1)$

$(A3)$ $Y(a,m_1,m_2)\perp M_2(e_2,m_1')$

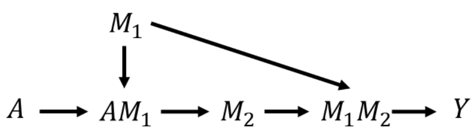
$(A4)$ $M_1(e_1)\perp A$

$(A5)$ $M_2(e_2,m_1)\perp\{A,M_1\}$

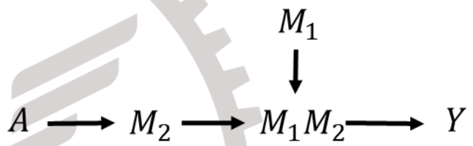
- $(A5.1)$ $M_2(e_2,m_1)\perp A$
- $(A5.2)$ $M_2(e_2,m_1)\perp M_1|A$

$(A6)$ $M_2(e_2,m_1)\perp M_1(e_1)$

</td><td>



</td></tr>
</table>

<table>
<tr><th colspan="2">$effect16$</th></tr>
<tr><th colspan="2">$Definition$</th></tr>
<tr><td colspan="2">$E\{[Y(0,1,1) - Y(0,1,0) - Y(0,0,1) + Y(0,0,0)][M_1(0)][M_2(1,0) - M_2(0,0)]\}$
$= E\big[Y\big(0, M_1(0), M_2(1,0)\big) - Y\big(0, M_1(0), M_2(0,0)\big)\big]$
$- E\big[Y\big(0,0, M_2(1,0)\big) - Y\big(0,0, M_2(0,0)\big)\big]$
$= [\phi_5(0,0,1,0) - \phi_5(0,0,0,0)] - [\phi_2(0,0,1,0) - \phi_2(0,0,0,0)]$
$= \Delta_5(0010 - 0000) - \Delta_2(0010 - 0000)$</td></tr>
<tr><th colspan="2">$Identification$</th></tr>
<tr><td colspan="2">$E\big[Y\big(a, M_1(e_1), M_2(e_2, m_1)\big)\big]$
$= \Sigma_{m_1 m_2} E[Y|A = a, M_1 = m_1, M_2 = m_2] P(M_2 = m_2|A = e_2, M_1 = m_1) P(M_1 = m_1|A = e_1)$
$E\big[Y\big(a, m_1, M_2(e_2, m_1')\big)\big]$
$= \Sigma_{m_2} E[Y|A = a, M_1 = m_1, M_2 = m_2] P(M_2 = m_2|A = e_2, M_1 = m_1')$</td></tr>
<tr><th>$Required\ Assumptions$</th><th>DAG</th></tr>
<tr><td>$(A1)\ Y(a, m_1, m_2) \perp \{A, M_1, M_2\}$
• $(A1.1)\ Y(a, m_1, m_2) \perp A$
• $(A1.2)\ Y(a, m_1, m_2) \perp M_1|A$
• $(A1.3)\ Y(a, m_1, m_2) \perp M_2|A, M_1$
$(A2)\ Y(a, m_1, m_2) \perp M_1(e_1)$
$(A3)\ Y(a, m_1, m_2) \perp M_2(e_2, m_1')$
$(A4)\ M_1(e_1) \perp A$
$(A5)\ M_2(e_2, m_1) \perp \{A, M_1\}$
• $(A5.1)\ M_2(e_2, m_1) \perp A$
• $(A5.2)\ M_2(e_2, m_1) \perp M_1|A$
$(A6)\ M_2(e_2, m_1) \perp M_1(e_1)$</td><td>
</td></tr>
</table>

effect17

Definition

$$E\{[Y(0,1,1)-Y(0,1,0)-Y(0,0,1)+Y(0,0,0)][M_1(0)-M_1(0)][M_2(1,1)-M_2(1,0)-M_2(0,1) + M_2(0,0)]\}$$

$$= \left\{E\left[Y\left(0,M_1(1),M_2\big(1,M_1(1)\big)\right)-Y\left(0,M_1(1),M_2\big(0,M_1(1)\big)\right)\right] - E\big[Y\big(0,M_1(1),M_2(1,0)\big)-Y\big(0,M_1(0),M_2(1,0)\big)\big] - E\big[Y\big(0,0,M_2(1,1)\big)-Y\big(0,0,M_2(0,1)\big)\big]\right\} - \left\{E\left[Y\left(0,M_1(0),M_2\big(1,M_1(0)\big)\right)-Y\left(0,M_1(0),M_2\big(0,M_1(0)\big)\right)\right] - E\big[Y\big(0,M_1(1),M_2(0,0)\big)-Y\big(0,M_1(0),M_2(0,0)\big)\big] - E\left[Y\left(0,0,M_2\big(1,M_1(0)\big)\right)-Y\left(0,0,M_2\big(0,M_1(0)\big)\right)\right]\right\}$$

$$= \{[\phi_6(0,1,1,1)-\phi_6(0,1,0,1)]-[\phi_5(0,1,1,0)-\phi_5(0,0,1,0)]-[\phi_3(0,0,1,1)-\phi_3(0,0,0,1)]\} - \{[\phi_6(0,0,1,0)-\phi_6(0,0,0,0)]-[\phi_5(0,1,0,0)-\phi_5(0,0,0,0)] - [\phi_3(0,0,1,0)-\phi_3(0,0,0,0)]\}$$

$$= [\Delta_6(0111-0101)-\Delta_5(0110-0010)-\Delta_3(0011-0001)]-[\Delta_6(0010-0000) - \Delta_5(0100-0000)-\Delta_3(0010-0000)]$$

Identification

$$E\left[Y\left(a,M_1(e_1),M_2\big(e_2,M_1(e_1)\big)\right)\right]$$

$$=\Sigma_{m_1m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1)P(M_1=m_1|A=e_1)$$

$$E\big[Y\big(a,M_1(e_1),M_2(e_2,m_1)\big)\big]$$

$$=\Sigma_{m_1m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1)P(M_1=m_1|A=e_1)$$

$$E\left[Y\left(a,m_1,M_2\big(e_2,M_1(e_1)\big)\right)\right]$$

$$=\Sigma_{m_1m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1)P(M_1=m_1|A=e_1)$$

Required Assumptions

$(A1)\ Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$
- $(A1.1)\ Y(a,m_1,m_2)\perp A$
- $(A1.2)\ Y(a,m_1,m_2)\perp M_1|A$
- $(A1.3)\ Y(a,m_1,m_2)\perp M_2|A,M_1$

$(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$

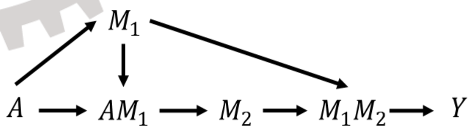
$(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$

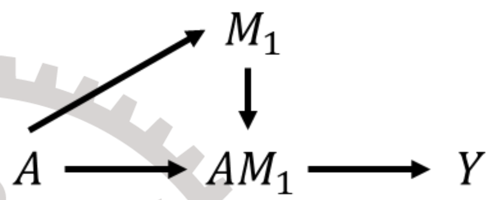
$(A4)\ M_1(e_1)\perp A$

$(A5)\ M_2(e_2,m_1)\perp\{A,M_1\}$
- $(A5.1)\ M_2(e_2,m_1)\perp A$
- $(A5.2)\ M_2(e_2,m_1)\perp M_1|A$

$(A6)\ M_2(e_2,m_1)\perp M_1(e_1)$

DAG



<table>
<tr><th colspan="2">$effect18$</th></tr>
<tr><td colspan="2">$Definition$</td></tr>
<tr><td colspan="2">$E\{[Y(1,1,0) - Y(1,0,0) - Y(0,1,0) + Y(0,0,0)][M_1(1) - M_1(0)]\}$
$= E[Y(1, M_1(1), 0) - Y(0, M_1(1), 0)] - E[Y(1, M_1(0), 0) - Y(0, M_1(0), 0)]$
$= [\phi_4(1,1,0) - \phi_4(0,1,0)] - [\phi_4(1,0,0) - \phi_4(0,0,0)]$
$= \Delta_4(110 - 010) - \Delta_4(100 - 000)$</td></tr>
<tr><td colspan="2">$Identification$</td></tr>
<tr><td colspan="2">$E[Y(a, M_1(e_1), m_2)]$
$= \Sigma_{m_1} E[Y|A = a, M_1 = m_1, M_2 = m_2] P(M_1 = m_1|A = e_1)$</td></tr>
<tr><td>$Required\ Assumptions$</td><td>DAG</td></tr>
<tr><td>$(A1)\ Y(a, m_1, m_2) \perp \{A, M_1, M_2\}$
• $(A1.1)\ Y(a, m_1, m_2) \perp A$
• $(A1.2)\ Y(a, m_1, m_2) \perp M_1|A$
• $(A1.3)\ Y(a, m_1, m_2) \perp M_2|A, M_1$
$(A2)\ Y(a, m_1, m_2) \perp M_1(e_1)$
$(A4)\ M_1(e_1) \perp A$</td><td></td></tr>
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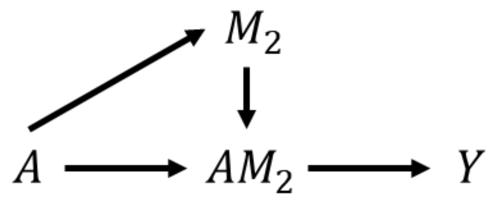
<table>
<tr><th colspan="2">$effect19$</th></tr>
<tr><td colspan="2">$Definition$</td></tr>
<tr><td colspan="2">$E\{[Y(1,0,1) - Y(1,0,0) - Y(0,0,1) + Y(0,0,0)][M_2(1,0) - M_2(0,0)]\}$
$= E[Y(1,0, M_2(1,0)) - Y(1,0, M_2(0,0))] - E[Y(0,0, M_2(1,0)) - Y(0,0, M_2(0,0))]$
$= [\phi_2(1,0,1,0) - \phi_2(1,0,0,0)] - [\phi_2(0,0,1,0) - \phi_2(0,0,0,0)]$
$= \Delta_2(1010 - 1000) - \Delta_2(0010 - 0000)$</td></tr>
<tr><td colspan="2">$Identification$</td></tr>
<tr><td colspan="2">$E[Y(a, m_1, M_2(e_2, m_1'))]$
$= \Sigma_{m_2} E[Y|A = a, M_1 = m_1, M_2 = m_2] P(M_2 = m_2|A = e_2, M_1 = m_1')$</td></tr>
<tr><td>$Required\ Assumptions$</td><td>DAG</td></tr>
<tr><td>$(A1)\ Y(a, m_1, m_2) \perp \{A, M_1, M_2\}$
• $(A1.1)\ Y(a, m_1, m_2) \perp A$
• $(A1.2)\ Y(a, m_1, m_2) \perp M_1|A$
• $(A1.3)\ Y(a, m_1, m_2) \perp M_2|A, M_1$
$(A3)\ Y(a, m_1, m_2) \perp M_2(e_2, m_1')$
$(A5)\ M_2(e_2, m_1) \perp \{A, M_1\}$
• $(A5.1)\ M_2(e_2, m_1) \perp A$
• $(A5.2)\ M_2(e_2, m_1) \perp M_1|A$</td><td></td></tr>
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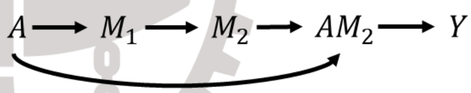
<table>
<tr><th colspan="2">$effect20$</th></tr>
<tr><td colspan="2" align="center">$Definition$</td></tr>
<tr><td colspan="2">$E\{[Y(1,0,1)-Y(1,0,0)-Y(0,0,1)+Y(0,0,0)][M_1(1)-M_1(0)][M_2(0,1)-M_2(0,0)]\}$
$=E\left[Y\left(1,0,M_2\left(0,M_1(1)\right)\right)-Y\left(1,0,M_2\left(0,M_1(0)\right)\right)\right]$
$-E\left[Y\left(0,0,M_2\left(0,M_1(1)\right)\right)-Y\left(0,0,M_2\left(0,M_1(0)\right)\right)\right]$
$=[\phi_3(1,0,0,1)-\phi_3(1,0,0,0)]-[\phi_3(0,0,0,1)-\phi_3(0,0,0,0)]$
$=\Delta_3(1001-1000)-\Delta_3(0001-0000)$</td></tr>
<tr><td colspan="2" align="center">$Identification$</td></tr>
<tr><td colspan="2">$E\left[Y\left(a,m_1,M_2\left(e_2,M_1(e_1)\right)\right)\right]$
$=\Sigma_{m_1m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1)P(M_1=m_1|A=e_1)$</td></tr>
<tr><td align="center">$Required\ Assumptions$</td><td align="center">DAG</td></tr>
<tr><td>$(A1)\ Y(a,m_1,m_2)\perp\{A\ ,M_1,M_2\}$
• $(A1.1)\ Y(a,m_1,m_2)\perp A$
• $(A1.2)\ Y(a,m_1,m_2)\perp M_1|A$
• $(A1.3)\ Y(a,m_1,m_2)\perp M_2|A,M_1$
$(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$
$(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$
$(A4)\ M_1(e_1)\perp A$
$(A5)\ M_2(e_2,m_1)\perp\{A,M_1\}$
• $(A5.1)\ M_2(e_2,m_1)\perp A$
• $(A5.2)\ M_2(e_2,m_1)\perp M_1|A$
$(A6)\ M_2(e_2,m_1)\perp M_1(e_1)$</td><td>
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$effect21$

$Definition$

$$E\{[Y(1,1,1) - Y(1,1,0) - Y(1,0,1) + Y(1,0,0) - Y(0,1,1) + Y(0,1,0) + Y(0,0,1) - Y(0,0,0)][M_1(1) - M_1(0)][M_2(0,0)]\}$$

$$= E\left[Y\big(1, M_1(1), M_2(0,0)\big) - Y\big(1, M_1(0), M_2(0,0)\big)\right] - E\left[Y\big(0, M_1(1), M_2(0,0)\big) - Y\big(0, M_1(0), M_2(0,0)\big)\right] - E[Y(1, M_1(1), 0) - Y(0, M_1(1), 0)] + E[Y(1, M_1(0), 0) - Y(0, M_1(0), 0)]$$

$$= [\phi_5(1,1,0,0) - \phi_5(1,0,0,0)] - [\phi_5(0,1,0,0) - \phi_5(0,0,0,0)] - [\phi_4(1,1,0) - \phi_4(0,1,0)] + [\phi_4(1,0,0) - \phi_4(0,0,0)]$$

$$= \Delta_5(1100 - 0100) - \Delta_5(0100 - 0000) - \Delta_4(110 - 010) + \Delta_4(100 - 000)$$

$Identification$

$$E\left[Y\big(a, M_1(e_1), M_2(e_2, m_1)\big)\right]$$

$$= \Sigma_{m_1 m_2} E[Y|A = a, M_1 = m_1, M_2 = m_2] P(M_2 = m_2|A = e_2, M_1 = m_1) P(M_1 = m_1|A = e_1)$$

$$E[Y(a, M_1(e_1), m_2)]$$

$$= \Sigma_{m_1} E[Y|A = a, M_1 = m_1, M_2 = m_2] P(M_1 = m_1|A = e_1)$$

$Required\ Assumptions$

$(A1)$ $Y(a, m_1, m_2) \perp \{A, M_1, M_2\}$

- $(A1.1)$ $Y(a, m_1, m_2) \perp A$
- $(A1.2)$ $Y(a, m_1, m_2) \perp M_1|A$
- $(A1.3)$ $Y(a, m_1, m_2) \perp M_2|A, M_1$

$(A2)$ $Y(a, m_1, m_2) \perp M_1(e_1)$

$(A3)$ $Y(a, m_1, m_2) \perp M_2(e_2, m_1')$

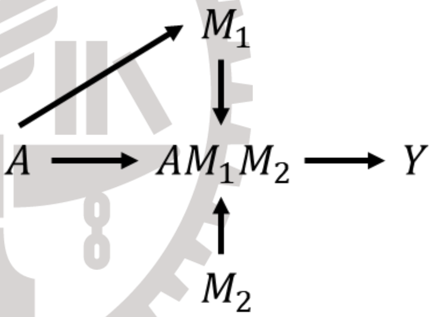
$(A4)$ $M_1(e_1) \perp A$

$(A5)$ $M_2(e_2, m_1) \perp \{A, M_1\}$

- $(A5.1)$ $M_2(e_2, m_1) \perp A$
- $(A5.2)$ $M_2(e_2, m_1) \perp M_1|A$

$(A6)$ $M_2(e_2, m_1) \perp M_1(e_1)$

DAG



<table>
<tr><th colspan="2">$effect22$</th></tr>
<tr><th colspan="2">$Definition$</th></tr>
<tr><td colspan="2">

$$
\begin{aligned}
&E\{[Y(1,1,1)-Y(1,1,0)-Y(1,0,1)+Y(1,0,0)-Y(0,1,1)+Y(0,1,0)+Y(0,0,1)\\
&\qquad -Y(0,0,0)][M_1(1)-M_1(0)][M_2(0,1)-M_2(0,0)]\}\\
&=\left\{E\left[Y\left(1,M_1(1),M_2\left(0,M_1(1)\right)\right)-Y\left(1,M_1(0),M_2\left(0,M_1(0)\right)\right)\right]\right.\\
&\qquad -E\left[Y\left(1,M_1(1),M_2(0,0)\right)-Y\left(0,M_1(1),M_2(0,0)\right)\right]\\
&\qquad \left.-E\left[Y\left(1,0,M_2\left(0,M_1(1)\right)\right)-Y\left(1,0,M_2\left(0,M_1(0)\right)\right)\right]\right\}\\
&\qquad -\left\{E\left[Y\left(0,M_1(1),M_2\left(0,M_1(1)\right)\right)-Y\left(0,M_1(0),M_2\left(0,M_1(0)\right)\right)\right]\right.\\
&\qquad -E\left[Y\left(0,M_1(1),M_2(0,0)\right)-Y\left(0,M_1(0),M_2(0,0)\right)\right]\\
&\qquad \left.-E\left[Y\left(0,0,M_2\left(0,M_1(1)\right)\right)-Y\left(0,0,M_2\left(0,M_1(0)\right)\right)\right]\right\}\\
&=\{[\phi_6(1,1,0,1)-\phi_6(1,0,0,0)]-[\phi_5(1,1,0,0)-\phi_5(0,1,0,0)]-[\phi_3(1,0,0,1)-\phi_3(1,0,0,0)]\}\\
&\qquad -\{[\phi_6(0,1,0,1)-\phi_6(0,0,0,0)]-[\phi_5(0,1,0,0)-\phi_5(0,0,0,0)]\\
&\qquad -[\phi_3(0,0,0,1)-\phi_3(0,0,0,0)]\}\\
&=[\Delta_6(1101-1000)-\Delta_5(1100-0100)-\Delta_3(1001-1000)]-[\Delta_6(0101-0000)\\
&\qquad -\Delta_5(0100-0000)-\Delta_3(0001-0000)]
\end{aligned}
$$

</td></tr>
<tr><th colspan="2">$Identification$</th></tr>
<tr><td colspan="2">

$E\left[Y\left(a,M_1(e_1),M_2\left(e_2,M_1(e_1)\right)\right)\right]$

$=\Sigma_{m_1m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1)P(M_1=m_1|A=e_1)$

$E\left[Y\left(a,M_1(e_1),M_2(e_2,m_1)\right)\right]$

$=\Sigma_{m_1m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1)P(M_1=m_1|A=e_1)$

$E\left[Y\left(a,m_1,M_2\left(e_2,M_1(e_1)\right)\right)\right]$

$=\Sigma_{m_1m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1)P(M_1=m_1|A=e_1)$

</td></tr>
<tr><th>$Required\ Assumptions$</th><th>DAG</th></tr>
<tr><td>

$(A1)\ Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$

- $(A1.1)\ Y(a,m_1,m_2)\perp A$
- $(A1.2)\ Y(a,m_1,m_2)\perp M_1|A$
- $(A1.3)\ Y(a,m_1,m_2)\perp M_2|A,M_1$

$(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$

$(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$

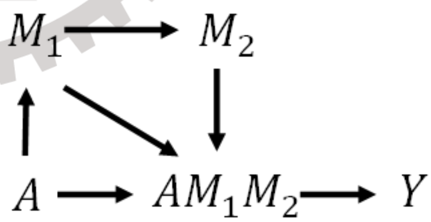
$(A4)\ M_1(e_1)\perp A$

$(A5)\ M_2(e_2,m_1)\perp\{A,M_1\}$

- $(A5.1)\ M_2(e_2,m_1)\perp A$
- $(A5.2)\ M_2(e_2,m_1)\perp M_1|A$

$(A6)\ M_2(e_2,m_1)\perp M_1(e_1)$

</td><td>



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</table>

<table>
<tr><th colspan="2">$effect23$</th></tr>
<tr><th colspan="2">$Definition$</th></tr>
<tr><td colspan="2">

$$E\{[Y(1,0,1)-Y(1,0,0)-Y(0,0,1)+Y(0,0,0)][M_1(1)-M_1(0)][M_2(1,1)-M_2(1,0)-M_2(0,1)+M_2(0,0)]\}$$

$$=\left\{E\left[Y\left(1,0,M_2(1,M_1(1))\right)-Y\left(1,0,M_2(1,M_1(0))\right)\right]-E\left[Y\left(1,0,M_2(0,M_1(1))\right)-Y\left(1,0,M_2(0,M_1(0))\right)\right]\right\}-\left\{E\left[Y\left(0,0,M_2(1,M_1(1))\right)-Y\left(0,0,M_2(1,M_1(0))\right)\right]-E\left[Y\left(0,0,M_2(0,M_1(1))\right)-Y\left(0,0,M_2(0,M_1(0))\right)\right]\right\}$$

$$=\{[\phi_3(1,0,1,1)-\phi_3(1,0,1,0)]-[\phi_3(1,0,0,1)-\phi_3(1,0,0,0)]\}-\{[\phi_3(0,0,1,1)-\phi_3(0,0,1,0)]-[\phi_3(0,0,0,1)-\phi_3(0,0,0,0)]\}$$

$$=[\Delta_3(1011-1010)-\Delta_3(1001-1000)]-[\Delta_3(0011-0010)-\Delta_3(0001-0000)]$$

</td></tr>
<tr><th colspan="2">$Identification$</th></tr>
<tr><td colspan="2">

$$E\left[Y\left(a,m_1,M_2(e_2,M_1(e_1))\right)\right]$$
$$=\Sigma_{m_1m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1)P(M_1=m_1|A=e_1)$$

</td></tr>
<tr><th>$Required\ Assumptions$</th><th>DAG</th></tr>
<tr><td>

$(A1)\ Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$

- $(A1.1)\ Y(a,m_1,m_2)\perp A$
- $(A1.2)\ Y(a,m_1,m_2)\perp M_1|A$
- $(A1.3)\ Y(a,m_1,m_2)\perp M_2|A,M_1$

$(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$

$(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$

$(A4)\ M_1(e_1)\perp A$

$(A5)\ M_2(e_2,m_1)\perp\{A,M_1\}$

- $(A5.1)\ M_2(e_2,m_1)\perp A$
- $(A5.2)\ M_2(e_2,m_1)\perp M_1|A$

$(A6)\ M_2(e_2,m_1)\perp M_1(e_1)$

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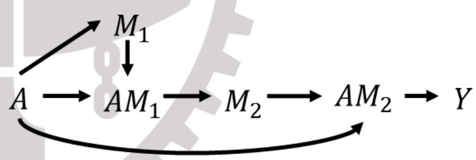

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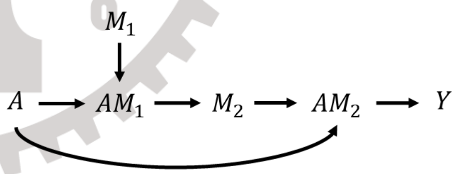
<table>
<tr><th colspan="2">$effect24$</th></tr>
<tr><td colspan="2" align="center">$Definition$</td></tr>
<tr><td colspan="2">$E\{[Y(1,0,1) - Y(1,0,0) - Y(0,0,1) + Y(0,0,0)][M_1(0)][M_2(1,1) - M_2(1,0) - M_2(0,1) + M_2(0,0)]\}$
$= \left\{E\left[Y\left(1,0,M_2(1,M_1(0))\right) - Y\left(1,0,M_2(0,M_1(0))\right)\right] - E\left[Y\left(1,0,M_2(1,0)\right) - Y\left(1,0,M_2(0,0)\right)\right]\right\} - \left\{E\left[Y\left(0,0,M_2(1,M_1(0))\right) - Y\left(0,0,M_2(0,M_1(0))\right)\right] - E\left[Y\left(0,0,M_2(1,0)\right) - Y\left(0,0,M_2(0,0)\right)\right]\right\}$
$= \{[\phi_3(1,0,1,0) - \phi_3(1,0,0,0)] - [\phi_2(1,0,1,0) - \phi_2(1,0,0,0)]\} - \{[\phi_3(0,0,1,0) - \phi_3(0,0,0,0)] - [\phi_2(0,0,1,0) - \phi_2(0,0,0,0)]\}$
$= [\Delta_3(1010 - 1000) - \Delta_2(1010 - 1000)] - [\Delta_3(0010 - 0000) - \Delta_2(0010 - 0000)]$</td></tr>
<tr><td colspan="2" align="center">$Identification$</td></tr>
<tr><td colspan="2">$E\left[Y\left(a,m_1,M_2(e_2,M_1(e_1))\right)\right]$
$= \Sigma_{m_1 m_2} E[Y \mid A = a, M_1 = m_1, M_2 = m_2] P(M_2 = m_2 \mid A = e_2, M_1 = m_1) P(M_1 = m_1 \mid A = e_1)$
$E\left[Y\left(a,m_1,M_2(e_2,m_1')\right)\right]$
$= \Sigma_{m_2} E[Y \mid A = a, M_1 = m_1, M_2 = m_2] P(M_2 = m_2 \mid A = e_2, M_1 = m_1')$</td></tr>
<tr><td align="center">$Required\ Assumptions$</td><td align="center">DAG</td></tr>
<tr><td>$(A1)\ Y(a,m_1,m_2) \perp \{A, M_1, M_2\}$
• $(A1.1)\ Y(a,m_1,m_2) \perp A$
• $(A1.2)\ Y(a,m_1,m_2) \perp M_1 \mid A$
• $(A1.3)\ Y(a,m_1,m_2) \perp M_2 \mid A, M_1$
$(A2)\ Y(a,m_1,m_2) \perp M_1(e_1)$
$(A3)\ Y(a,m_1,m_2) \perp M_2(e_2,m_1')$
$(A4)\ M_1(e_1) \perp A$
$(A5)\ M_2(e_2,m_1) \perp \{A, M_1\}$
• $(A5.1)\ M_2(e_2,m_1) \perp A$
• $(A5.2)\ M_2(e_2,m_1) \perp M_1 \mid A$
$(A6)\ M_2(e_2,m_1) \perp M_1(e_1)$</td><td></td></tr>
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<table>
<tr><th colspan="2">$effect25$</th></tr>
<tr><th colspan="2">$Definition$</th></tr>
<tr><td colspan="2">

$$E\{[Y(1,1,1)-Y(1,1,0)-Y(1,0,1)+Y(1,0,0)-Y(0,1,1)+Y(0,1,0)+Y(0,0,1)-Y(0,0,0)][M_1(1)-M_1(0)][M_2(1,0)-M_2(0,0)]\}$$

$$=\{E[Y(1,M_1(1),M_2(1,0))-Y(1,M_1(0),M_2(1,0))]-E[Y(1,M_1(1),M_2(0,0))-Y(1,M_1(0),M_2(0,0))]\}-\{E[Y(0,M_1(1),M_2(1,0))-Y(0,M_1(0),M_2(1,0))]-E[Y(0,M_1(1),M_2(0,0))-Y(0,M_1(0),M_2(0,0))]\}$$

$$=\{[\phi_5(1,1,1,0)-\phi_5(1,0,1,0)]-[\phi_5(1,1,0,0)-\phi_5(1,0,0,0)]\}-\{[\phi_5(0,1,1,0)-\phi_5(0,0,1,0)]-[\phi_5(0,1,0,0)-\phi_5(0,0,0,0)]\}$$

$$=[\Delta_5(1110-1010)-\Delta_5(1100-1000)]-[\Delta_5(0110-0010)-\Delta_5(0100-0000)]$$

</td></tr>
<tr><th colspan="2">$Identification$</th></tr>
<tr><td colspan="2">

$$E[Y(a,M_1(e_1),M_2(e_2,m_1))]$$

$$=\Sigma_{m_1m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1)P(M_1=m_1|A=e_1)$$

</td></tr>
<tr><th>$Required\ Assumptions$</th><th>DAG</th></tr>
<tr><td>

$(A1)$ $Y(a,m_1,m_2)\perp\{A\ ,M_1,M_2\}$

- $(A1.1)$ $Y(a,m_1,m_2)\perp A$
- $(A1.2)$ $Y(a,m_1,m_2)\perp M_1|A$
- $(A1.3)$ $Y(a,m_1,m_2)\perp M_2|A,M_1$

$(A2)$ $Y(a,m_1,m_2)\perp M_1(e_1)$

$(A3)$ $Y(a,m_1,m_2)\perp M_2(e_2,m_1')$

$(A4)$ $M_1(e_1)\perp A$

$(A5)$ $M_2(e_2,m_1)\perp\{A,M_1\}$

- $(A5.1)$ $M_2(e_2,m_1)\perp A$
- $(A5.2)$ $M_2(e_2,m_1)\perp M_1|A$

$(A6)$ $M_2(e_2,m_1)\perp M_1(e_1)$

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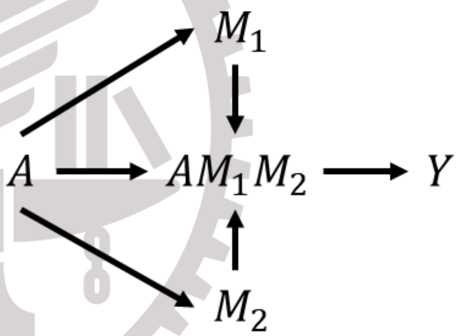

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$effect26$

$Definition$

$$E\{[Y(1,1,1) - Y(1,1,0) - Y(1,0,1) + Y(1,0,0) - Y(0,1,1) + Y(0,1,0) + Y(0,0,1) - Y(0,0,0)][M_1(1) - M_1(0)][M_2(1,1) - M_2(1,0) - M_2(0,1) + M_2(0,0)]\}$$

$$= \Big\{E\Big[Y\Big(1, M_1(1), M_2\big(1, M_1(1)\big)\Big) - Y\Big(1, M_1(1), M_2\big(0, M_1(1)\big)\Big)\Big] - E[Y(1, M_1(1), M_2(1,0)) - Y(1, M_1(1), M_2(0,0))]$$

$$- E\Big[Y\Big(1, M_1(0), M_2\big(1, M_1(0)\big)\Big) - Y\Big(1, M_1(0), M_2\big(0, M_1(0)\big)\Big)\Big]$$

$$+ E[Y(1, M_1(0), M_2(1,0)) - Y(1, M_1(0), M_2(0,0))] - E\Big[Y\Big(1,0, M_2\big(1, M_1(1)\big)\Big) - Y\Big(1,0, M_2\big(1, M_1(0)\big)\Big)\Big]$$

$$+ E\Big[Y\Big(1,0, M_2\big(0, M_1(1)\big)\Big) - Y\Big(1,0, M_2\big(0, M_1(0)\big)\Big)\Big]\Big\}$$

$$- \Big\{E\Big[Y\Big(0, M_1(1), M_2\big(1, M_1(1)\big)\Big) - Y\Big(0, M_1(1), M_2\big(0, M_1(1)\big)\Big)\Big]$$

$$- E[Y(0, M_1(1), M_2(1,0)) - Y(0, M_1(1), M_2(0,0))]$$

$$- E\Big[Y\Big(0, M_1(0), M_2\big(1, M_1(0)\big)\Big) - Y\Big(0, M_1(0), M_2\big(0, M_1(0)\big)\Big)\Big]$$

$$+ E[Y(0, M_1(0), M_2(1,0)) - Y(0, M_1(0), M_2(0,0))] - E\Big[Y\Big(0,0, M_2\big(1, M_1(1)\big)\Big) - Y\Big(0,0, M_2\big(1, M_1(0)\big)\Big)\Big]$$

$$+ E\Big[Y\Big(0,0, M_2\big(0, M_1(1)\big)\Big) - Y\Big(0,0, M_2\big(0, M_1(0)\big)\Big)\Big]\Big\}$$

$$= \{[\phi_6(1,1,1,1) - \phi_6(1,1,0,1)] - [\phi_5(1,1,1,0) - \phi_5(1,1,0,0)] - [\phi_6(1,0,1,0) - \phi_6(1,0,0,0)] + [\phi_5(1,0,1,0) - \phi_5(1,0,0,0)]$$

$$- [\phi_3(1,0,1,1) - \phi_3(1,0,1,0)] + [\phi_3(1,0,0,1) - \phi_3(1,0,0,0)]\} - \{[\phi_6(0,1,1,1) - \phi_6(0,1,0,1)]$$

$$- [\phi_5(0,1,1,0) - \phi_5(0,1,0,0)] - [\phi_6(0,0,1,0) - \phi_6(0,0,0,0)] + [\phi_5(0,0,1,0) - \phi_5(0,0,0,0)]$$

$$- [\phi_3(0,0,1,1) - \phi_3(0,0,1,0)] + [\phi_3(0,0,0,1) - \phi_3(0,0,0,0)]$$

$Identification$

$$E\Big[Y\Big(a, M_1(e_1), M_2\big(e_2, M_1(e_1)\big)\Big)\Big]$$

$$= \Sigma_{m_1 m_2} E[Y|A = a, M_1 = m_1, M_2 = m_2] P(M_2 = m_2|A = e_2, M_1 = m_1) P(M_1 = m_1|A = e_1)$$

$$E\big[Y\big(a, M_1(e_1), M_2(e_2, m_1)\big)\big]$$

$$= \Sigma_{m_1 m_2} E[Y|A = a, M_1 = m_1, M_2 = m_2] P(M_2 = m_2|A = e_2, M_1 = m_1) P(M_1 = m_1|A = e_1)$$

$$E\Big[Y\Big(a, m_1, M_2\big(e_2, M_1(e_1)\big)\Big)\Big]$$

$$= \Sigma_{m_1 m_2} E[Y|A = a, M_1 = m_1, M_2 = m_2] P(M_2 = m_2|A = e_2, M_1 = m_1) P(M_1 = m_1|A = e_1)$$

$Required\ Assumptions$

$(A1)$ $Y(a, m_1, m_2) \perp \{A, M_1, M_2\}$

- $(A1.1)$ $Y(a, m_1, m_2) \perp A$
- $(A1.2)$ $Y(a, m_1, m_2) \perp M_1|A$
- $(A1.3)$ $Y(a, m_1, m_2) \perp M_2|A, M_1$

$(A2)$ $Y(a, m_1, m_2) \perp M_1(e_1)$

$(A3)$ $Y(a, m_1, m_2) \perp M_2(e_2, m_1')$

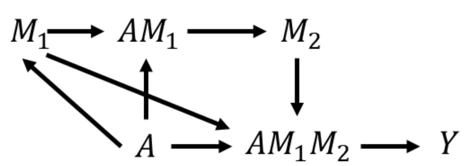
$(A4)$ $M_1(e_1) \perp A$

$(A5)$ $M_2(e_2, m_1) \perp \{A, M_1\}$

- $(A5.1)$ $M_2(e_2, m_1) \perp A$
- $(A5.2)$ $M_2(e_2, m_1) \perp M_1|A$

$(A6)$ $M_2(e_2, m_1) \perp M_1(e_1)$

DAG



effect27

Definition

$$E\{[Y(1,1,1)-Y(1,1,0)-Y(1,0,1)+Y(1,0,0)-Y(0,1,1)+Y(0,1,0)+Y(0,0,1)-Y(0,0,0)][M_1(0)][M_2(1,1)-M_2(1,0)-M_2(0,1)+M_2(0,0)]\}$$

$$=\Big\{E\Big[Y\Big(1,M_1(0),M_2\big(1,M_1(0)\big)\Big)-Y\Big(1,M_1(0),M_2\big(0,M_1(0)\big)\Big)\Big]-E\big[Y\big(1,M_1(0),M_2(1,0)\big)-Y\big(1,M_1(0),M_2(0,0)\big)\big]$$

$$-E\Big[Y\Big(1,0,M_2\big(1,M_1(0)\big)\Big)-Y\Big(1,0,M_2\big(0,M_1(0)\big)\Big)\Big]+E\big[Y\big(1,0,M_2(1,0)\big)-Y\big(1,0,M_2(0,0)\big)\big]\Big\}$$

$$-\Big\{E\Big[Y\Big(0,M_1(0),M_2\big(1,M_1(0)\big)\Big)-Y\Big(0,M_1(0),M_2\big(0,M_1(0)\big)\Big)\Big]$$

$$-E\big[Y\big(0,M_1(0),M_2(1,0)\big)-Y\big(0,M_1(0),M_2(0,0)\big)\big]-E\Big[Y\Big(0,0,M_2\big(1,M_1(0)\big)\Big)-Y\Big(0,0,M_2\big(0,M_1(0)\big)\Big)\Big]$$

$$+E\big[Y\big(0,0,M_2(1,0)\big)-Y\big(0,0,M_2(0,0)\big)\big]\Big\}$$

$$=\{[\phi_6(1,0,1,0)-\phi_6(1,0,0,0)]-[\phi_5(1,0,1,0)-\phi_5(1,0,0,0)]-[\phi_3(1,0,1,0)-\phi_3(1,0,0,0)]+[\phi_2(1,0,1,0)-\phi_2(1,0,0,0)]\}$$

$$-\{[\phi_6(0,0,1,0)-\phi_6(0,0,0,0)]-[\phi_5(0,0,1,0)-\phi_5(0,0,0,0)]-[\phi_3(0,0,1,0)-\phi_3(0,0,0,0)]+[\phi_2(0,0,1,0)-\phi_2(0,0,0,0)]$$

$$=[\Delta_6(1010-1000)-\Delta_5(1010-1000)-\Delta_3(1010-1000)+\Delta_2(1010-1000)]$$

$$-[\Delta_6(0010-0000)-\Delta_5(0010-0000)-\Delta_3(0010-0000)+\Delta_2(0010-0000)]$$

Identification

$E\Big[Y\Big(a,M_1(e_1),M_2\big(e_2,M_1(e_1)\big)\Big)\Big]$

$=\Sigma_{m_1m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1)P(M_1=m_1|A=e_1)$

$E\big[Y\big(a,M_1(e_1),M_2(e_2,m_1)\big)\big]$

$=\Sigma_{m_1m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1)P(M_1=m_1|A=e_1)$

$E\Big[Y\Big(a,m_1,M_2\big(e_2,M_1(e_1)\big)\Big)\Big]$

$=\Sigma_{m_1m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1)P(M_1=m_1|A=e_1)$

$E\big[Y\big(a,m_1,M_2(e_2,m_1')\big)\big]$

$=\Sigma_{m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1')$

Required Assumptions

$(A1)\ Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$

- $(A1.1)\ Y(a,m_1,m_2)\perp A$
- $(A1.2)\ Y(a,m_1,m_2)\perp M_1|A$
- $(A1.3)\ Y(a,m_1,m_2)\perp M_2|A,M_1$

$(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$

$(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$

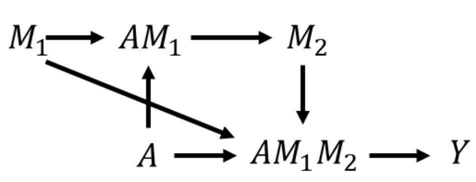
$(A4)\ M_1(e_1)\perp A$

$(A5)\ M_2(e_2,m_1)\perp\{A,M_1\}$

- $(A5.1)\ M_2(e_2,m_1)\perp A$
- $(A5.2)\ M_2(e_2,m_1)\perp M_1|A$

$(A6)\ M_2(e_2,m_1)\perp M_1(e_1)$

DAG



<table>
<tr><th colspan="2">$effect28$</th></tr>
<tr><td colspan="2" align="center">$Definition$</td></tr>
<tr><td colspan="2">

$$
\begin{aligned}
&E\{[Y(1,1,1)-Y(1,1,0)-Y(1,0,1)+Y(1,0,0)-Y(0,1,1)+Y(0,1,0)+Y(0,0,1)\\
&\qquad -Y(0,0,0)][M_1(0)][M_2(1,0)-M_2(0,0)]\}\\
&=\{E[Y(1,M_1(0),M_2(1,0))-Y(0,M_1(0),M_2(1,0))]\\
&\qquad -E[Y(1,0,M_2(1,0))-Y(1,0,M_2(0,0))]\}\\
&\qquad -\{E[Y(1,M_1(0),M_2(0,0))-Y(0,M_1(0),M_2(0,0))]\\
&\qquad -E[Y(0,0,M_2(1,0))-Y(0,0,M_2(0,0))]\}\\
&=\{[\phi_5(1,0,1,0)-\phi_5(0,0,1,0)]-[\phi_2(1,0,1,0)-\phi_2(1,0,0,0)]\}-\{[\phi_5(1,0,0,0)\\
&\qquad -\phi_5(0,0,0,0)]-[\phi_2(0,0,1,0)-\phi_2(0,0,0,0)]\}\\
&=[\Delta_5(1010-0010)-\Delta_2(1010-1000)]\\
&\qquad -[\Delta_5(1000-0000)-\Delta_2(0010-0000)]
\end{aligned}
$$

</td></tr>
<tr><td colspan="2" align="center">$Identification$</td></tr>
<tr><td colspan="2">

$E[Y(a,M_1(e_1),M_2(e_2,m_1))]$

$=\Sigma_{m_1m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1)P(M_1=m_1|A=e_1)$

$E[Y(a,m_1,M_2(e_2,m_1'))]$

$=\Sigma_{m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2,M_1=m_1')$

</td></tr>
<tr><td align="center">$Required\ Assumptions$</td><td align="center">DAG</td></tr>
<tr><td>

$(A1)\ Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$

- $(A1.1)\ Y(a,m_1,m_2)\perp A$
- $(A1.2)\ Y(a,m_1,m_2)\perp M_1|A$
- $(A1.3)\ Y(a,m_1,m_2)\perp M_2|A,M_1$

$(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$

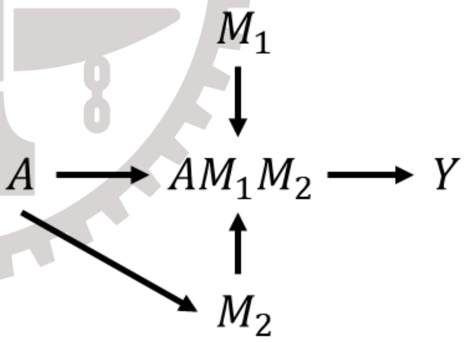
$(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$

$(A5)\ M_2(e_2,m_1)\perp\{A,M_1\}$

- $(A5.1)\ M_2(e_2,m_1)\perp A$
- $(A5.2)\ M_2(e_2,m_1)\perp M_1|A$

$(A6)\ M_2(e_2,m_1)\perp M_1(e_1)$

</td><td>



</td></tr>
</table>

2.3 二十八條路徑特定效應的中介路徑準則判斷

前幾節，簡述了二十八路徑拆解法的效應生成機制，並且藉由因果參數ϕ_1至ϕ_6定義二十八條路徑特定效應，也將定義結果拓展至因果圖，協助了解其因果效應的作用機制。不過，我們也好奇此二十八條路徑特定效應的定義，是否為合適的中介效應定義？

為此我們引入學者(Miles, 2023)所提出的中介路徑準則，該準則解釋中介效應機制於個體層次下的涵義為：

「(a)改變暴露因子A的介入導致中介因子M的改變

(b)中介因子M的變化將導致結果變量Y的變化」(Miles, 2023)

針對此中介效應的涵意，學者(Miles, 2023)提出了中介路徑準則，用以判斷什麼是正確的中介效應測量指標，並透過該指標判斷自然間接效應(natural indirect effect, NIE)和介入間接效應(interventional indirect effect, IIE)，最後得出自然間接效應是最符合中介路徑準則的中介測量指標。

關於該中介路徑準則的定義在論文中將其分為兩個部分，分別為虛無準則(null criteria)與單調性準則(monotonicity criteria)，以此小節我們將討論上述兩個準則並將其拓展至二十八路徑拆解法的二十八條路徑特定效應。

(2.3.1) 中介路徑準則—虛無準則的定義與判別方式

學者(Miles, 2023)對於虛無準則(null criteria)的概念，與假設檢定中的虛無假說(H_0)、對立假說(H_1)相互對應。然而，論文中並未精確的描述這種對應關係。因此，我們在以下定義中，統稱為「H_0準則」和「H_1準則」以便簡化理解。以下以數學形式呈現這兩個準則的條件定義：

單一中介因子情境下，對於每個個體i，

$$\begin{cases} H_0: Y_i(a',m) - Y_i(a',m') = 0 \cup M_i(a) - M_i(a') = 0 \\ H_1: Y_i(a',m) - Y_i(a',m') \neq 0 \cap M_i(a) - M_i(a') \neq 0 \end{cases} \quad ,\forall\, a, a', m, m'.$$

「H_0 準則」成立的情況是：在H_0條件之下，則該效應為零；反之，則不成

立。「H_1 準則」成立的情況是：H_1條件之下，則該效應不為零；反之，則不成立。上述的定義與準則判別方式，可以協助研究者釐清中介效應的理想定義。

而我們也試著將此中介路徑準則，拓展至雙有序中介因子的情況。理想上，會涵蓋暴露因子A對於結果變量Y的直接因果效應，$A \rightarrow Y$；暴露因子A通過中介因子M_1再對於結果變量Y的間接因果效應，$A \rightarrow M_1$ 、$M_1 \rightarrow Y$；暴露因子A通過中介因子M_2再對於結果變量Y的間接因果效應，$A \rightarrow M_2$ 、$M_2 \rightarrow Y$，以及暴露因子A通過中介因子M_1再通過中介因子M_2對於結果變量Y的間接因果效應，$A \rightarrow M_1$ 、$M_1 \rightarrow M_2$ 、$M_2 \rightarrow Y$，我們分別定義為「PSE_{00} 準則」、「PSE_{10} 準則」、「PSE_{02} 準則」、「PSE_{12}準則」，以下則是呈現這四個條件定義及判別準則。

雙有序中介因子情境下，對於每個個體 i

「PSE_{00}準則」

$$\begin{cases} H_0: Y_i(a', m_1', m_2') - Y_i(a'', m_1', m_2') = 0 \\ H_1: Y_i(a', m_1', m_2') - Y_i(a'', m_1', m_2') \neq 0 \end{cases}, \forall\, a', a'', m_1', m_2'$$

「PSE_{10}準則」

$$\begin{cases} H_0: M_{1i}(a') - M_{1i}(a'') = 0 \cup Y_i(a', m_1', m_2') - Y_i(a', m_1'', m_2') = 0 \\ H_1: M_{1i}(a') - M_{1i}(a'') \neq 0 \cap Y_i(a', m_1', m_2') - Y_i(a', m_1'', m_2') \neq 0 \end{cases}, \forall\, a', a'', m_1', m_1'', m_2'$$

「PSE_{02}準則」

$$\begin{cases} H_0: M_{2i}(a', m_1') - M_{2i}(a'', m_1') = 0 \cup Y_i(a', m_1', m_2') - Y_i(a', m_1', m_2'') = 0 \\ H_1: M_{2i}(a', m_1') - M_{2i}(a'', m_1') \neq 0 \cap Y_i(a', m_1', m_2') - Y_i(a'', m_1', m_2'') \neq 0 \end{cases}, \forall\, a', a'', m_1', m_2', m_2''$$

「PSE_{12}準則」

$$\begin{cases} H_0: M_{1i}(a') - M_{1i}(a'') = 0 \cup M_{2i}(a', m_1') - M_{2i}(a', m_1'') = 0 \cup Y_i(a', m_1', m_2') - Y_i(a', m_1', m_2'') = 0 \\ H_1: M_{1i}(a') - M_{1i}(a'') \neq 0 \cap M_{2i}(a', m_1') - M_{2i}(a', m_1'') \neq 0 \cap Y_i(a', m_1', m_2') - Y_i(a'', m_1', m_2'') \neq 0 \end{cases}$$

$$\forall\, a', a'', m_1', m_1'', m_2', m_2''$$

判別準則為：在H_0條件之下，則該效應為零；反之，則不成立。在H_1條件之下，則該效應不為零；反之，則不成立。

上述的定義與判別準則主要是以數學式的方式呈現，若是以較為不嚴謹但是直觀的解釋方式，則是將反事實模型的定義以因果圖的方式呈現，其結果如下：

「PSE_{00}準則」

$$\begin{cases} H_0: A \to Y = 0 \\ H_1: A \to Y \neq 0 \end{cases}$$

「PSE_{10}準則」

$$\begin{cases} H_0: A \to M_1 = 0 \ \cup M_1 \to Y = 0 \\ H_1: A \to M_1 \neq 0 \ \cap \ M_1 \to Y \neq 0 \end{cases}$$

「PSE_{02}準則」

$$\begin{cases} H_0: A \to M_2 = 0 \ \cup \ M_2 \to Y = 0 \\ H_1: A \to M_2 \neq 0 \ \cap \ M_2 \to Y \neq 0 \end{cases}$$

「PSE_{12}準則」

$$\begin{cases} H_0: A \to M_1 = 0 \ \cup M_1 \to M_2 = 0 \ \cup \ M_2 \to Y = 0 \\ H_1: A \to M_1 \neq 0 \ \cap M_1 \to M_2 \neq 0 \cap M_2 \to Y \neq 0 \end{cases}$$

(2.3.2) 中介路徑準則的判別結果

透過前一小節的定義與判別準則，我們將判斷二十八條路徑特定效應，是否為合適的中介效應，由上述的準則判別也可以發現，「H_1準則」通過的條件會比「H_0準則」還要嚴格，若「H_0準則」成立可能不見得「H_1準則」也會成立；若「H_1準則」成立，則「H_0準則」必定要成立。

而準則成立，表示此路徑特定效應，可以被分類出此效應的產生是經由什麼路徑所產生的，例如：某效應在「PSE_{10}準則」的「H_0 準則」成立，則表示此效應可被歸類於$A \to M_1$ 、$M_1 \to Y$的路徑。最理想的情況下，希望H_0 、H_1都同時成立，但如果只有H_0成立的話，也可以被分類於其中一個效應路徑，只是數學條件上不夠嚴謹。以下則是透過定義與因果圖判別二十八條路徑特定效應是否有通過中介路徑準則的判別結果，灰色部分表示該效應成立，呈現於表 2.7。

表 2.7 二十八條路徑特定效應通過中介路徑準則的判別結果

	PSE_{00}準則		PSE_{10}準則		PSE_{02}準則		PSE_{12}準則	
	H_0	H_1	H_0	H_1	H_0	H_1	H_0	H_1
$effect1$								
$effect2$								
$effect3$								
$effect4$								
$effect5$								
$effect6$								
$effect7$								
$effect8$								
$effect9$								
$effect10$								
$effect11$								
$effect12$								
$effect13$								
$effect14$								
$effect15$								
$effect16$								
$effect17$								
$effect18$								
$effect19$								
$effect20$								
$effect21$								
$effect22$								
$effect23$								
$effect24$								
$effect25$								
$effect26$								
$effect27$								
$effect28$								

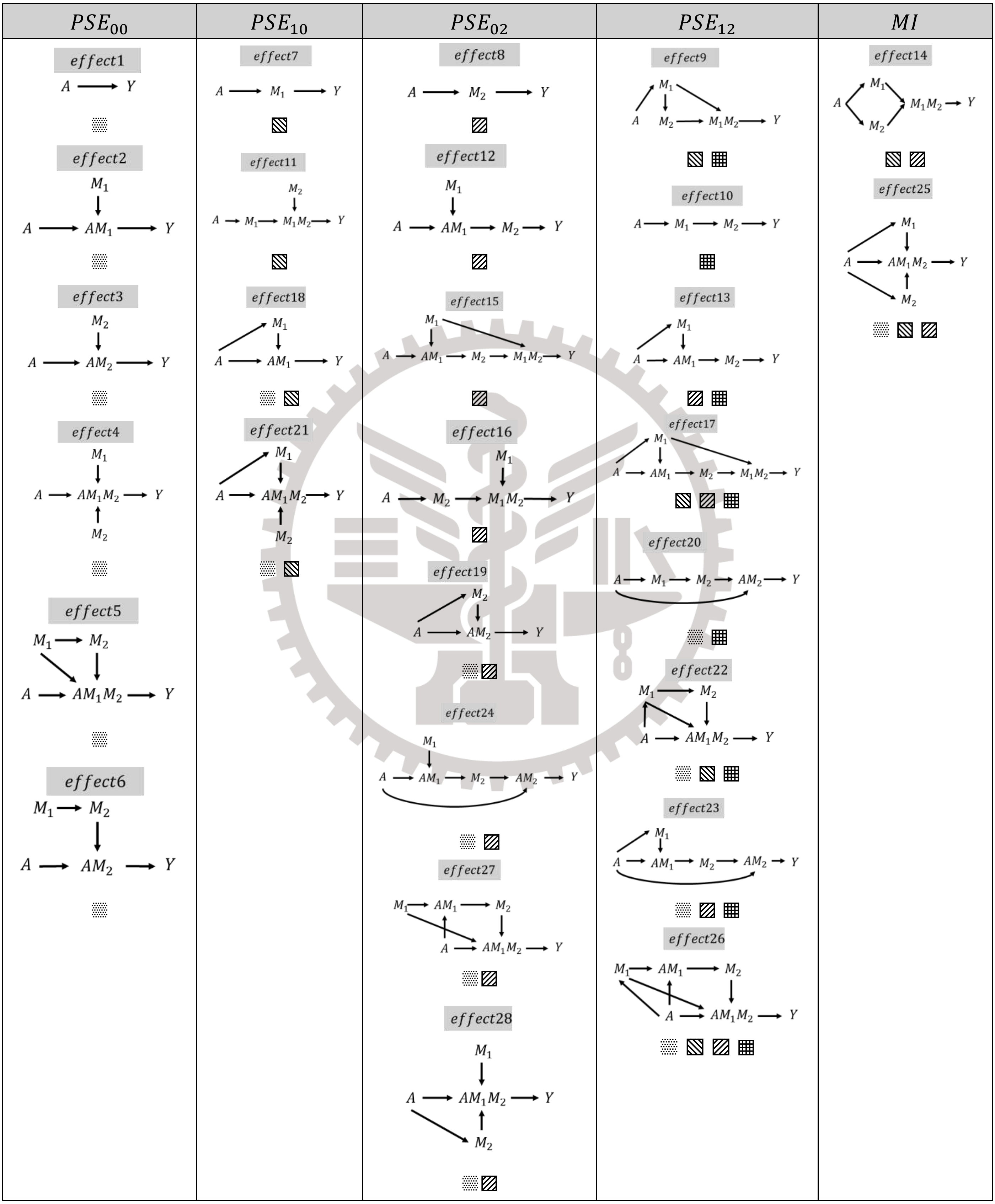
傳統上，雙有序中介因子的部分順向拆解法，會將路徑效應區分成PSE_{00}、PSE_{10}、PSE_{02}、PSE_{12}，分別代表未通過任何中介因子的效應$(A \to Y)$、僅通過M_1的效應$(A \to M_1 \to Y)$、僅通過M_2的效應$(A \to M_2 \to Y)$、先通過M_1再通過M_2的效應$(A \to M_1 \to M_2 \to Y)$，若是我們將二十八條路徑特定效應與其對照，便會發現，二十八路徑拆解法所拆解出的路徑效應，並無法透過傳統的雙有序中介因子的效應路徑進行分類，進行分類的結果便會發現同一個效應，有可能同時通過不同的中介因子所產生。而同時通過「H_0 準則」、「H_1 準則」的效應僅有$effect1$、$effect7$、$effect8$、$effect10$，從因果圖也可以很直觀的理解是符合傳統的PSE_{00}、PSE_{10}、PSE_{02}、PSE_{12}的定義。除此之外，其餘的效應主要只有「H_0 準則」成立而已。

除了上述的四條效應分類，主要是以暴露因子A先通過的中介因子進行區分，但是從二十八條特定路徑效應的因果圖中，我們可以發現有些效應是暴露因子A同時通過中介因子M_1、M_2的情形，例如$effect14$、$effect25$。我們將引入新的分類(mediated interaction, MI)，用以表示該效應的中介路徑。我們將這五條路徑進行分類並整理在表 2.8，主要通過的中介路徑準則為「H_0 準則」。由於有同時通過不同路徑的情形，在此我們將定義不同的中介路徑準則的有效程度，將PSE_{12}視為最主要的中介路徑，其次是PSE_{10}、PSE_{02}兩條路徑分類的有效程度是相近的，最後則是PSE_{00}。舉例來說：$effect26$同時通過「H_0 準則」中四條路徑準則，則PSE_{12}是最主要的中介路徑，接著是PSE_{10}、PSE_{02}，最後才是PSE_{00}。

以下則是我們將上述的分類準則方式，透過表格進行整理，以協助判定二十八路徑效應的分類情況，便於審視效應通過中介路徑的方式。

表 2.8 二十八條路徑特定效應的分類

▒表示同時通過PSE_{00} ▧表示同時通過PSE_{10} ▨表示同時通過PSE_{02} ▦表示同時通過PSE_{12}



由表 2.7、2.8 我們可以發現，二十八條路徑特定效應產生的路徑，並不是僅有一個路徑可以分類的，舉例來說：$effect26$，分別通過「H_0 準則」中四條路徑準則，表示$effect26$涵蓋暴露因子A對於結果變量Y的直接因果效應，$A \to Y$；暴露因子A通過中介因子M_1再對於結果變量Y的間接因果效應，$A \to M_1$ 、$M_1 \to Y$；暴露因子A通過中介因子M_2再對於結果變量Y的間接因果效應，$A \to M_2$ 、$M_2 \to Y$，以及暴露因子A通過中介因子M_1再通過中介因子M_2對於結果變量Y的間接因果效應，$A \to M_1$ 、$M_1 \to M_2$ 、$M_2 \to Y$，雖然可以從中了解，$effect26$可能包含的路徑效應，但是卻無法提供研究者有效的分類資訊。

對此我們懷疑傳統上分類中介路徑效應的方式是否正確？而學者(Miles, 2023)所提出的中介效應路徑準則，可能存在著需要被修正的地方。

2.4 因果參數、辨識假設的彙整

綜觀前一小節對於過去學者提出的拆解法之回顧，我們可以發現無論是平行中介、有序中介因子的拆解法，都可以使用$effect1$~$effect28$表達，而$effect1$~$effect28$則都可以寫成因果參數ϕ_1~ϕ_6的線性組合，若是從數學式所代表的機制，我們可以發現，透過改變結果變量Y、中介因子M_1 、M_2的反事實模型中的a 、m_1 、m_2變量，可以呈現該數學式所表示的效應，例如：$effect1$ 為A對於Y的直接影響，在結果變量Y的反事實模型中，暴露因子A寫成$a = 1$ 、$a = 0$，表示為$E[Y(1,0,0) - Y(0,0,0)]$，可以整理成因果參數$\phi_1(1,0,0) - \phi_1(0,0,0)$，此時，我們定義$\phi_1(1,0,0) - \phi_1(0,0,0)$為$\phi_1(\Delta a, 0,0)$；$effect25$的因果參數表達式為$\{[\phi_5(1,1,1,0) - \phi_5(1,0,1,0)] - [\phi_5(1,1,0,0) - \phi_5(1,0,0,0)]\} - \{[\phi_5(0,1,1,0) - \phi_5(0,0,1,0)] - [\phi_5(0,1,0,0) - \phi_5(0,0,0,0)]\}$，我們可以將其簡化成：$\phi_5(\Delta a, \Delta e_1, \Delta e_2, 0)$，此表示法可以更直觀的提供我們該效應的變化量為何。

當我們以變化量的方式呈現因果參數時，也能將其視為二十八條路徑效應的組合，例如：$\phi_3(\Delta a, 0,0,0)$可以表示成$effect1$ 、$effect3$ 、$effect6$的加總，以下表格則是我們將二十八條路徑效應，與因果參數ϕ_1至ϕ_6變化量的彙整，同時與辨識假設對應，協助研究者未來能更直觀的理解此二十八條路徑效應於因果參數的變化量所產生的效應，並且透過中介效應準則的判讀結果，協助釐清該效應屬於哪一個中介路徑的分類。

表 2.9 二十八條路徑特定效應與中介效應準則判讀結果與辨識假設的彙整

		effect																											
PSE Criteria		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
PSE_{00}	H_0	■	■	■	■	■	■												■	■	■	■	■	■	■	■	■	■	■
PSE_{10}	H_0							■		■		■			■			■	■			■				■	■		
PSE_{02}	H_0								■				■	■	■	■	■	■		■				■	■	■	■	■	■
PSE_{12}	H_0									■	■			■				■			■		■	■			■		
Causal Parameters		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
$\phi_1(\Delta a,0,0)$		■																											
$\phi_2(\Delta a,0,0,0)$		■		■																									
$\phi_2(\Delta a,0,\Delta e_2,0)$																				■									
$\phi_2(0,0,\Delta e_2,0)$									■																				
$\phi_3(\Delta a,0,0,0)$		■		■			■																						
$\phi_3(\Delta a,0,\Delta e_2,0)$																				■					■				
$\phi_3(0,0,0,\Delta e_3)$											■																		
$\phi_3(\Delta a,0,0,\Delta e_3)$																					■								
$\phi_3(0,0,\Delta e_2,0)$									■				■																
$\phi_3(\Delta a,0,\Delta e_2,\Delta e_3)$																								■					
$\phi_3(0,0,\Delta e_2,\Delta e_3)$														■															
$\phi_4(\Delta a,\Delta e_1,0)$																			■										
$\phi_5(\Delta a,0,\Delta e_2,0)$																				■									■
$\phi_5(\Delta a,\Delta e_1,0,0)$																			■			■							
$\phi_5(0,\Delta e_1,0,0)$								■				■																	
$\phi_5(0,0,\Delta e_2,0)$									■								■												
$\phi_5(0,\Delta e_1,\Delta e_2,0)$															■														
$\phi_5(\Delta a,\Delta e_1,\Delta e_2,0)$																										■			
$\phi_6(\Delta a,0,0,0)$		■	■	■	■	■	■																						
$\phi_6(0,\Delta e_1,0,\Delta e_3)$								■		■	■	■																	
$\phi_6(0,0,\Delta e_2,0)$									■				■			■	■												
$\phi_6(0,\Delta e_1,\Delta e_2,\Delta e_3)$														■	■			■											
$\phi_6(\Delta a,\Delta e_2,0,\Delta e_3)$																			■		■	■	■						
$\phi_6(\Delta a,1,\Delta e_2,1)$																				■				■	■	■	■	■	■

Assumptions	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
(A1.1)	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
(A1.2)	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
(A1.3)	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
(A2)		■		■	■	■	■		■	■	■	■	■	■	■	■	■	■		■	■	■	■	■	■	■	■	■
(A3)			■	■	■	■		■	■	■	■	■	■	■	■	■	■		■	■	■	■	■	■	■	■	■	■
(A4)		■		■	■	■	■		■	■	■	■	■	■	■	■	■	■		■	■	■	■	■	■	■	■	■
(A5.1)			■	■	■	■		■	■	■	■	■	■	■	■	■	■		■	■	■	■	■	■	■	■	■	■
(A5.2)			■	■	■	■		■	■	■	■	■	■	■	■	■	■		■	■	■	■	■	■	■	■	■	■
(A6)				■	■	■			■	■	■	■	■	■	■	■	■			■	■	■	■	■	■	■	■	■

下一節我們將平均因果效應尺度的二十八路徑拆解法拓展至族群介入效應(population intervention effect, PIE)的尺度。這將有助於我們評估介入措施的整體效果以及不同中介機制的重要性。我們期望能夠提供更全面的視角，分析介入措施在真實世界中的潛在影響，並為公共衛生或政策制定提供更精確的資訊。

第三章、雙中介因子之族群介入效應的機制分析

族群可歸因分率(population attributable fraction, PAF)是公共衛生研究中，用於評估暴露因子對結果變量影響程度的重要指標。它衡量的是：若完全移除某個暴露因子，特定疾病的發生率可望降低的比例。以下我們討論當暴露因子A與特定疾病Y均為二元變量時的數學表達式：

$$PAF = \frac{Pr(Y=1) - Pr\,(Y=1|A=0)}{Pr\,(Y=1)}$$

- $Pr\,(Y=1)$：表示特定疾病Y的發生率。
- $Pr\,(Y=1|A=0)$：表示特定疾病Y在未暴露於暴露因子A的疾病發生率。

此公式的計算結果能幫助研究者量化暴露因子對特定疾病的影響程度，進而評估減少暴露所能帶來的公共衛生效益。

然而，上述公式僅適用於單一暴露因子的簡化情境，難以完整描述現實中複雜的公共衛生問題。隨著方法學的發展，PAF 分析逐漸導入因果推論架構，並考慮A、Y二元變量的情況，疾病的發生率等於期望值，意即$Pr(Y=1) = E[Y]$、$Pr(Y(0)=1) = E[Y(0)]$。在反事實模型框架下，PAF 可進一步表示為：

$$PAF = \frac{E[Y - Y(0)]}{E[Y]}$$

接著我們引入學者(Hubbard&Van der Laan, 2008)所提出的族群介入效應(population intervention effect, PIE)，其效應的數學式定義如下：

$$PIE = E[Y - Y(0)]$$

在公共衛生的領域中，$E[Y]$代表的是特定疾病Y感染人數之期望值，$E[Y(0)]$則是於特定疾病Y於反事實模型中移除暴露因子A的期望值，PIE 的定義為上述兩個期望值的對比，可以協助研究者評估公共衛生干預措施的潛在影響，以及處理有害暴露因子的觀察性研究中，提供了一種有效的因果推論方法，與 PAF 尺度相比之下僅在於分母需除以$E[Y]$，是屬於真實世界的結果變量Y的期望值，所以 PIE 與 PAF 尺度所使用的反事實模型基本上是相同的。

此章節我們將回顧 PAF、PIE 路徑拆解方法，並且將雙有序中介因子中介效應與交互作用的分析拓展至 PIE 的尺度，發展 PIE 三十三路徑拆解法的最細部拆解，並詳細說明因果參數，進一步探討辨識假設及其對效應辨識的影響，呈現 PIE 三十三路徑拆解法下的辨識結果，並與先前學者提出的拆解方法進行比較，期望為 PIE 尺度下雙有序中介因子的研究提供拓展的可能。

3.1 回顧因果框架下的 PIE、PAF 的中介效應分析

(3.1.1) Sjölander之 PAF 二路徑拆解法

學者(Sjölander, 2018)將 PAF 的尺度引入中介效應分析的框架，將 PAF 進行兩路徑的拆解，提出有通過中介因子 M 的自然直接可歸因分率(natural direct AF, NDAF)以及沒有通過中介因子M的自然間接可歸因分率(natural indirect AF, NIAF)，其計算方式分別為：

$$NDAF = \frac{E[Y - Y(0, M)]}{E[Y]}$$

$$NIAF = \frac{E[Y(0, M) - Y(0)]}{E[Y]}$$

在效應的詮釋上，如果暴露因子A對於結果變量Y的直接效應等於0，那就表示即便我們阻斷暴露因子A對於結果變量Y所造成的效應，也不會改變結果變量Y所發生的機率，即$E[Y - Y(0, M)] = 0$，導致$\mathrm{NDAF} = 0$；而暴露因子A對於結果變量Y的中介效應等於0，那就表示即便我們阻斷A對於M所造成的效應，也不會改變特定疾病Y所發生的機率，即$E[Y(0, M) - Y(0)]$，導致$\mathrm{NDAF} = 0$。關於上述效應的解釋，反映出學者(Sjölander, 2018)提出之 PAF 二路徑拆解法與傳統上中介效應分析的自然直接效應(natural direct effect, NDE)、自然間接效應(natural direct effect, NIE)的詮釋一致，提供研究者較為直觀的因果效應解釋。

(3.1.2) Fulcher et al.之 PIE 二路徑拆解法

而學者(Fulcher et. al, 2020)則將 PIE 拓展至單中介因子的二路徑拆解法，將 PIE 分解為有通過中介因子 M 的族群介入直接效應(population intervention direct effect, PIDE)、沒有通過中介因子M的族群介入中介效應(population intervention indirect effect, PIIE)，分別定義為：

$$PIE = E[Y - Y(0)] = PIDE + PIIE$$

$$PIDE = E\big[Y\big(A, M(0)\big) - Y(0)\big]$$

$$PIIE = E\big[Y - Y\big(A, M(0)\big)\big]$$

上述的拆解法和學者(Sjölander, 2018)同樣是將效應拆解為兩個路徑，但兩位學者的效應詮釋卻有所差異：NDAF 將中介因子M設定為現實世界的觀測值，

PIDE 則是將中介因子M控制在$M(0)$的情況下去比較暴露因子A的改變對於結果變量Y所造成的影響；NIAF 則是將暴露因子A=0的情況下去做討論，PIIE 則是將暴露因子A設定為現實世界的觀測值，去比較中介因子M對於結果變量Y造成的效應。

(3.1.3) O′Connell多重中介因子 PAF 拆解法

學者(O′Connell, 2022) 則提出了一種新的估計 PAF 尺度的方法，稱為特定路徑效應可歸因分率 (pathway − specific PAF, PS − PAF)。該方法分別考量介入型(interventional)、機制型(mechanistic)和分解型(separable)的路徑效應，並且，上述三類的路徑效應，都各自有在因果推論上的解釋意義，其假設在多重平行中介因子的情況下討論，$M_1, M_2, \ldots, M_K$為暴露因子A到結果變量Y的因果路徑中所經過的K個中介因子。

機制型 PS-PAF 描述的是，如果完全移除某條中介因果途徑，例如：$A \rightarrow M_i \rightarrow Y$($M_i$表示第$i$個中介因子，$1 \leqq i \leqq K$)，那麼在整個族群上，結果變量$Y$的發生機率會減少多少。這種方法可以被理解為，在一個假設的世界中，某個中介因子M_i維持在沒有暴露因子A的狀態，而其他因素不變，來觀察疾病的變化。這種方法適用於當我們可以清楚識別因果途徑，並評估是否可以透過直接干預該途徑來減少結果變量Y的發生機率。

介入型 PS-PAF 則是假設對整個族群進行干預，使某個中介因子M_i服從假設沒有暴露因子A的機率分佈，然後測量結果變量Y發生機率的變化。這種方法可以視為一種「隨機介入」，即讓整個族群的中介因子機率分佈變成沒有暴露因子A的情況，而不直接干預風險因子本身。這種方法適用於公共衛生介入措施對結果變量Y的發生機率影響。

分解型 PS-PAF 則是使用學者(Robins et. al, 2010)提出的可分解路徑框架，該框架是假設暴露因子A的影響可以拆分為兩個「可獨立控制」的兩個部分，一個部分A_{M^k}表示暴露因子A經過中介因子M_k對於結果變量Y造成的路徑效應，另一個部分A_{O^k}代表暴露因子A直接影響結果變量Y或是經過其他中介因子M_j但不經過$M_k (j \neq k)$間接影響Y的路徑效應。這種方法要求我們能夠找到一種介入方式，使得可以獨立暴露因子A對不同中介因子的影響，而不影響其他因果途徑。以下我們將上述三種不同類型的 PS-PAF，並整理數學式與反事實模型的解釋。

表 3.1 三種類型的 PS-PAF 之數學式與其解釋

類型	PS-PAF 定義之數學式	數學式的解釋
介入型	$PAF^{I}_{A\to M^k\to Y}=\frac{E\left[Y-Y\left(A,G^k(0\|C)\right)\right]}{E[Y]}$	$E[Y]$代表結果變量Y發生機率，$G^k(0\|C)$代表在個體干擾因子C的情況下，從移除暴露因子A的第k個中介因子$M_k(0)$的條件分布中隨機抽樣的值。
機制型	$PAF^{M}_{A\to M^k\to Y}=\frac{E\left[Y-Y\left(A,M_k(0)\right)\right]}{E[Y]}$	$E[Y]$代表結果變量Y發生機率，$M_k(0)$代表第k個中介因子M_k在$A=a$時的反事實模型。
分解型	$PAF^{S}_{A\to M^k\to Y}=\frac{E\left[Y-\left(Y\middle\|do\left(A_{M^k}=0\right)\right)\right]}{E[Y]}$	$E[Y]$代表結果變量Y發生機率，A_{M^k}代表A經過中介因子M^k對於Y所造成的路徑效應，$do(A_{M^k}=0)$代表介入A_{M^k}的值使之等於 0，改變了暴露因子A在群體中的分布狀態。

(3.1.4) 本研究室 PAF 五路徑拆解法因果參數與假設

回顧本研究室(Duan, 2024)提出的 PAF 五路徑拆解方法，並且依據 PAF 及 PIE 的關係，以下我們將使用 PIE 的尺度進行討論，藉此描述單一中介因子之族群介入效應於中介效應與交互作用可能的路徑。這些參數以巢狀反事實模型(nested counterfactual model)為基礎，涵蓋暴露因子A、中介因子M與結果變量Y的組合情境，定義為族群介入效應的尺度。

表 3.2 單一中介因子的 PIE 之因果參數定義

因果參數反事實模型定義	反事實模型解釋
$E[Y(a)]$	暴露因子$A=a$，結果變量Y的反事實模型。
$E[Y(a,m)]$	暴露因子$A=a$、中介因子$M=m$，結果變量Y的反事實模型。
$E[Y(A,m')]$	暴露因子A為現實觀測值，中介因子$M=m'$，結果變量Y的反事實模型。
$E[Y(a',M)]$	暴露因子$A=a'$，中介因子M為現實觀測值，結果變量Y的反事實模型。
$E\left[Y\left(A,M(a')\right)\right]$	暴露因子A為現實觀測值，中介因子$M(a')$時，結果變量Y的反事實模型。

上述的定義是建構於反事實模型的框架底下，因此我們需要在某些假設的前提下，進行因果參數的辨識，下列$(A1)$~$(A4)$則分別表達暴露因子A、中介因子M、結果變量Y之間的未測量干擾因子(unmeasured confounding)、時變干擾因子(time − varing confounding)，以下將$A \to Y$的未測量干擾因子定義為U_{AY}、$M \to Y$的未測量干擾因子定義為U_{MY}、$A \to M$的未測量干擾因子定義為U_{AM}、時變干擾因子為L，以下將介紹考慮基線干擾因子C的辨識假設以及辨識結果。

$(A1)$ $Y(a,m) \perp A|C$，暴露因子A與結果變量Y之間不應存在未測量干擾因子U_{AY}。

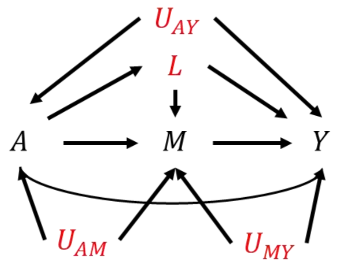
$(A2)$ $Y(a,m) \perp M|\{A,C\}$，中介因子M與結果變量Y之間不應存在未測量干擾因子U_{MY}。

$(A3)$ $Y(a,m) \perp M(a')|C$，中介因子M與結果變量Y之間不應存在未測量干擾因子U_{MY}、時變干擾因子L。

$(A4)$ $M(a) \perp A|C$，暴露因子A與中介因子M之間不應存在未測量干擾因子U_{AM}。

表 3.3 單一中介因子 PIE 之因果參數辨識結果與使用假設

<table>
<tr><th>因果參數</th><th colspan="5">辨識結果</th></tr>
<tr><td>$E[Y(a)]$</td><td colspan="5">$\Sigma_c E[Y|A=a,C=c]P(C=c)$</td></tr>
<tr><td>$E[Y(a,m)]$</td><td colspan="5">$\Sigma_c E[Y|A=a,M=m,C=c]P(C=c)$</td></tr>
<tr><td>$E[Y(A,m')]$</td><td colspan="5">$\Sigma_{a,c} E[Y|A=a,M=m',C=c]P(A=a|C=c)P(C=c)$</td></tr>
<tr><td>$E[Y(a',M)]$</td><td colspan="5">$\Sigma_{a,m,c} E[Y|A=a',M=m,C=c]P(M=m|A=a,C=c)P(A=a|C=c)P(C=c)$</td></tr>
<tr><td>$E\left[Y\left(A,M(a')\right)\right]$</td><td colspan="5">$\Sigma_{a,m,c} E[Y|A=a,M=m,C=c]P(M=m|A=a',C=c)P(A=a|C=c)P(C=c)$</td></tr>
<tr><th colspan="6">辨識假設</th></tr>
<tr><td colspan="6">$(A1)$ $Y(a,m) \perp A|C$
$(A2)$ $Y(a,m) \perp M|\{A,C\}$
$(A3)$ $Y(a,m) \perp M(a')|C$
$(A4)$ $M(a) \perp A|C$</td></tr>
<tr><td></td><td>$(A1)$</td><td>$(A2)$</td><td>$(A3)$</td><td>$(A4)$</td><td>不應存在之干擾因子</td></tr>
<tr><td>$E[Y(a)]$</td><td style="background:#d3d3d3"></td><td></td><td></td><td></td><td>U_{AY}</td></tr>
<tr><td>$E[Y(a,m)]$</td><td style="background:#d3d3d3"></td><td></td><td style="background:#d3d3d3"></td><td></td><td>U_{AY}、U_{MY}</td></tr>
<tr><td>$E[Y(A,m')]$</td><td></td><td></td><td style="background:#d3d3d3"></td><td></td><td>U_{MY}</td></tr>
<tr><td>$E[Y(a',M)]$</td><td style="background:#d3d3d3"></td><td style="background:#d3d3d3"></td><td style="background:#d3d3d3"></td><td></td><td>U_{AY}、U_{MY}、L</td></tr>
<tr><td>$E\left[Y\left(A,M(a')\right)\right]$</td><td style="background:#d3d3d3"></td><td style="background:#d3d3d3"></td><td style="background:#d3d3d3"></td><td style="background:#d3d3d3"></td><td>U_{AY}、U_{MY}、U_{AM}、L</td></tr>
</table>

不應存在之干擾因子	$(A1)$	$(A2)$	$(A3)$	$(A4)$	
	U_{AY}	U_{MY}	U_{MY} L	U_{AM}	

為便於理解單一中介因子的中介效應與交互作用的機制分析，我們引入其因果圖，如圖 3.1 所示，我們可以發現單一中介因子之中介效應與交互作用的分析機制需考量暴露因子A、中介因子M、交互作用項AM、結果變量Y的組合。本研究室(Duan, 2024)將生成因果效應的暴露因子、交互作用項、結果變量的組合一一整理。

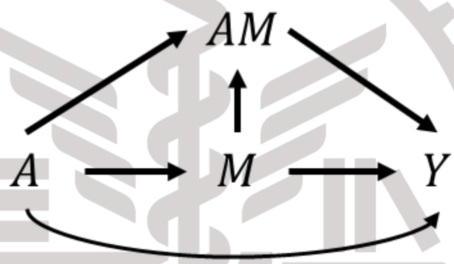


圖 3.1 單一中介因子下的中介效應與交互作用因果圖

透過單一中介因子下的因果圖，可以較為直觀的理解效應的生成機制，像是總共有三個箭頭指向結果變量Y，對於Y的成因可以區分為三種生成方式：(1)A生成、(2)M生成、(3)AM交互生成。而中介因子M則會出現「自行生成」效應或是「永不生成」的情況，表示效應有無經過暴露因子A的情況，所以中介因子M的生成方式可以分為(1)A生成、(2)自行生成、(3)永不生成。透過上述的窮舉以及組合的方式，可以協助研究者得出 PIE 的五路徑拆解法的效應。接著下一節我們將單一中介因子的中介效應與交互作用分析，拓展至雙有序中介因子，定義其因果參數、辨識假設、呈現辨識結果。

表 3.4 單一中介因子生成機制整理

效應	M生成方式	Y生成方式	因果參數的定義
PIE_{CDE}	永不生成	A生成	$E[Y(A,0)-Y(0,0)]$
$PIE_{INT_{ref}}$	自行生成	AM交互生成	$E\big[Y\big(A,M(0)\big)-Y\big(0,M(0)\big)-\big(Y(A,0)-Y(0,0)\big)\big]$
$PIE_{INT_{med}}$	A生成	AM交互生成	$E\Big[Y-Y(0,M)-\Big(Y\big(A,M(0)\big)-Y(0)\Big)\Big]$
PIE_{PIE}	A生成	M生成	$E[Y(0,M)-Y(0)]$
PIE_{CEM}	自行生成	M生成	$E\big[Y\big(0,M(0)\big)-Y(0,0)\big]$

3.2 雙有序中介因子 PIE 因果參數、假設與辨識結果

(3.2.1) 定義 PIE 尺度下延伸之因果參數

在 PIE 尺度下的因果參數與第二章討論的平均因果效應ϕ_1至 ϕ_6之間的主要差異在於暴露因子 A 的考量方式。平均因果效應中的因果參數ϕ_1 至 ϕ_6 假設暴露因子 A是介入的常數，而在 PIE 尺度下，則進一步考量了 A在現實世界中的實際觀測值。因此，PIE 尺度下的因果參數不僅包含 ϕ_1 至 ϕ_6，還會基於這些參數進一步調整，以反映暴露因子 A 在真實世界中的情況。舉例來說：$\phi_1 = E[Y(a',m_1',m_2')]$，暴露因子 $A=a'$於 PIE 尺度下，會出現$\phi_7 = E[Y(A,m_1',m_2')]$的因果參數；$\phi_2 = E\big[Y\big(a',m_1',M_2(e_2',m_1')\big)\big]$，暴露因子 $A=a'$、$M_2(e_2',m_1')$於 PIE 尺度下，會出現$\phi_8 = E\big[Y\big(A,m_1',M_2(e_2',m_1')\big)\big]$、$\phi_9 = E\big[Y\big(a',m_1',M_2(A,m_1')\big)\big]$、$\phi_{10} = E\big[Y\big(A,m_1',M_2(A,m_1')\big)\big]$的因果參數，以下表 3.5 則是呈現因果參數的數目，以及之後所延伸的因果參數，延伸的因果參數數目為2^k-1，$k\equiv$現實世界觀測值A表示的位置，唯一不同的是ϕ_6會產生$E\Big[Y\Big(A,M_1(A),M_2\big(A,M_1(A)\big)\Big)\Big]$的情況，此時的因果參數不需要使用辨識假設，$E\Big[Y\Big(A,M_1(A),M_2\big(A,M_1(A)\big)\Big)\Big]=E[Y]$，所以延伸因果參數的數目計算上需要多減去該參數，除了ϕ_1至ϕ_6之外，還有二十七個延伸因果參數，總共有三十三個因果參數。

表 3.5 雙有序中介因子 PIE 延伸之因果參數數目

因果參數		反事實模型中以現實世界觀測值A表示	延伸因果參數數目	延伸的因果參數
ϕ_1	$E[Y(a',m_1',m_2')]$	$Y(a',m_1',m_2')$	$2^1-1=1$	ϕ_7
ϕ_2	$E[Y(a',m_1',M_2(e_2',m_1'))]$	$Y(a',m_1',M_2(e_2',m_1'))$	$2^2-1=3$	ϕ_8~ϕ_{10}
ϕ_3	$E\left[Y\left(a',m_1',M_2(e_2',M_1(e_1'))\right)\right]$	$Y\left(a',m_1',M_2(e_2',M_1(e_1'))\right)$	$2^3-1=7$	ϕ_{11}~ϕ_{17}
ϕ_4	$E[Y(a',M_1(e_1'),m_2')]$	$Y(a',M_1(e_1'),m_2')$	$2^2-1=3$	ϕ_{18}~ϕ_{20}
ϕ_5	$E[Y(a',M_1(e_1'),M_2(e_2',m_1'))]$	$Y(a',M_1(e_1'),M_2(e_2',m_1'))$	$2^3-1=7$	ϕ_{21}~ϕ_{27}
ϕ_6	$E\left[Y\left(a',M_1(e_1'),M_2(e_2',M_1(e_1'))\right)\right]$	$Y\left(a',M_1(e_1'),M_2(e_2',M_1(e_1'))\right)$	$2^3-2=6$	ϕ_{28}~ϕ_{33}

(3.2.2) 因果參數的辨識假設與辨識結果

由於 PIE 尺度下的因果參數是因果參數ϕ_1至ϕ_6的延伸，所以辨識過程所使用的假設也和因果參數ϕ_1至ϕ_6相同，其差異在於 PIE 尺度下的因果參數可以使用較少的辨識假設，以下則是表 3.6、3.7 則是將定義 PIE 尺度下延伸之因果參數、辨識使用假設、假設辨識的結果、不應存在之干擾因子、ϕ_7至ϕ_{33}可以省略的假設進行整理。

表 3.6 雙有序中介因子辨識假設整理

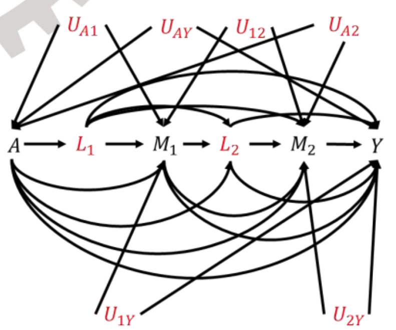
辨識假設	不應存在之干擾因子	不應存在之干擾因子因果圖
$(A1.1)\ Y(a,m_1,m_2)\perp A$	U_{AY}	
$(A1.2)\ Y(a,m_1,m_2)\perp M_1\|A$	U_{1Y}	
$(A1.3)\ Y(a,m_1,m_2)\perp M_2\|A$、$M_1$	U_{2Y}	
$(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$	U_{1Y}、L_1	
$(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$	U_{2Y}、L_1、L_2	
$(A4)\ M_1(e_1)\perp A$	U_{A1}	
$(A5.1)\ M_2(e_2,m_1)\perp A$	U_{A2}	
$(A5.2)\ M_2(e_2,m_1)\perp M_1\|A$	U_{12}	
$(A6)\ M_2(e_2,m_1)\perp M_1(e_1)$	U_{12}、L_1	

表 3.7 雙有序中介因子 PIE 因果參數辨識假設、結果、可以省略的假設

分類	因果參數	反事實模型定義	辨識結果	
ϕ_1	$\phi_1(a', m_1', m_2')$	$E[Y(a', m_1', m_2')]$	$E[Y\|A = a', M_1 = m_1', M_2 = m_2']$	
ϕ_2	$\phi_2(a', m_1', e_2', m_1')$	$E[Y(a', m_1', M_2(e_2', m_1'))]$	$\Sigma_{m_2} E[Y\|A = a', M_1 = m_1', M_2 = m_2] P(M_2 = m_2\|A = e_2', M_1 = m_1')$	
ϕ_3	$\phi_3(a', m_1', e_2', e_1')$	$E\left[Y\left(a', m_1', M_2(e_2', M_1(e_1'))\right)\right]$	$\Sigma_{m_1,m_2} E[Y\|A = a', M_1 = m_1', M_2 = m_2]$ $P(M_2 = m_2\|A = e_2', M_1 = m_1) P(M_1 = m_1\|A = e_1')$	
ϕ_4	$\phi_4(a', e_1', m_2')$	$E[Y(a', M_1(e_1'), m_2')]$	$\Sigma_{m_1} E[Y\|A = a', M_1 = m_1, M_2 = m_2'] P(M_1 = m_1\|A = e_1')$	
ϕ_5	$\phi_5(a', e_1', e_2', m_1')$	$E[Y(a', M_1(e_1'), M_2(e_2', m_1'))]$	$\Sigma_{m_1,m_2} E[Y\|A = a', M_1 = m_1, M_2 = m_2]$ $P(M_2 = m_2\|A = e_2', M_1 = m_1') P(M_1 = m_1\|A = e_1')$	
ϕ_6	$\phi_6(a', e_1', e_2', e_1')$	$E\left[Y\left(a', M_1(e_1'), M_2(e_2', M_1(e_1'))\right)\right]$	$\Sigma_{m_1,m_2} E[Y\|A = a', M_1 = m_1, M_2 = m_2]$ $P(M_2 = m_2\|A = e_2', M_1 = m_1) P(M_1 = m_1\|A = e_1')$	
分類	因果參數	反事實模型定義	辨識結果	可以省略的辨識假設
ϕ_1	$\phi_7(A, m_1', m_2')$	$E[Y(A, m_1', m_2')]$	$\Sigma_a E[Y\|A = a, M_1 = m_1', M_2 = m_2'] P(A = a)$	$(A1.1)$
ϕ_2	$\phi_8(A, m_1', e_2', m_1')$	$E[Y(A, m_1', M_2(e_2', m_1'))]$	$\Sigma_{a,m_2} E[Y\|A = a, M_1 = m_1', M_2 = m_2]$ $P(M_2 = m_2\|A = e_2', M_1 = m_1') P(A = a)$	$(A1.1)$
	$\phi_9(a', m_1', A, m_1')$	$E[Y(a', m_1', M_2(A, m_1'))]$	$\Sigma_{a,m_2} E[Y\|A = a', M_1 = m_1', M_2 = m_2]$ $P(M_2 = m_2\|A = a, M_1 = m_1') P(A = a)$	$(A5.1)$
	$\phi_{10}(A, m_1', A, m_1')$	$E[Y(A, m_1', M_2(A, m_1'))]$	$\Sigma_{a,m_2} E[Y\|A = a, M_1 = m_1', M_2 = m_2]$ $P(M_2 = m_2\|A = a, M_1 = m_1') P(A = a)$	$(A1.1)$、$(A3)$、$(A5.1)$
ϕ_3	$\phi_{11}(a', m_1', e_2', M_1)$	$E[Y(a', m_1', M_2(e_2', M_1))]$	$\Sigma_{a,m_1,m_2} E[Y\|A = a', M_1 = m_1', M_2 = m_2] P(M_2 = m_2\|A = e_2', M_1 = m_1)$ $P(M_1 = m_1\|A = a) P(A = a)$	$(A4)$
	$\phi_{12}(A, m_1', e_2', M_1)$	$E[Y(A, m_1', M_2(e_2', M_1))]$	$\Sigma_{a,m_1,m_2} E[Y\|A = a, M_1 = m_1', M_2 = m_2]$ $P(M_2 = m_2\|A = e_2', M_1 = m_1) P(M_1 = m_1\|A = a) P(A = a)$	$(A1.1)$、$(A2)$、$(A4)$
	$\phi_{13}(a', m_1', A, M_1)$	$E[Y(a', m_1', M_2(A, M_1))]$	$\Sigma_{a,m_1,m_2} E[Y\|A = a', M_1 = m_1', M_2 = m_2] P(M_2 = m_2\|A = a, M_1 = m_1)$ $P(M_1 = m_1\|A = a) P(A = a)$	$(A4)$、$(A5.1)$、$(A5.2)$、$(A6)$
	$\phi_{14}(A, m_1', A, M_1)$	$E[Y(A, m_1', M_2(A, M_1))]$	$\Sigma_{a,m_1,m_2} E[Y\|A = a, M_1 = m_1', M_2 = m_2] P(M_2 = m_2\|A = a, M_1 = m_1)$ $P(M_1 = m_1\|A = a) P(A = a)$	$(A1.1)$、$(A2)$、$(A4)$、$(A5.1)$、$(A5.2)$、$(A6)$
	$\phi_{15}(A, m_1', e_2', e_1')$	$E\left[Y\left(A, m_1', M_2(e_2', M_1(e_1'))\right)\right]$	$\Sigma_{a,m_1,m_2} E[Y\|A = a, M_1 = m_1', M_2 = m_2] P(M_2 = m_2\|A = e_2', M_1 = m_1) P(M_1 = m_1\|A = e_1') P(A = a)$	$(A1.1)$
	$\phi_{16}(a', m_1', A, e_1')$	$E\left[Y\left(a', m_1', M_2(A, M_1(e_1'))\right)\right]$	$\Sigma_{a,m_1,m_2} E[Y\|A = a', M_1 = m_1', M_2 = m_2] P(M_2 = m_2\|A = a, M_1 = m_1)$ $P(M_1 = m_1\|A = e_1') P(A = a)$	$(A5.1)$
	$\phi_{17}(A, m_1', A, e_1')$	$E\left[Y\left(A, m_1', M_2(A, M_1(e_1'))\right)\right]$	$\Sigma_{a,m_1,m_2} E[Y\|A = a, M_1 = m_1', M_2 = m_2] P(M_2 = m_2\|A = a, M_1 = m_1)$ $P(M_1 = m_1\|A = e_1') P(A = a)$	$(A1.1)$、$(A5.1)$

分類	因果參數	辨識假設									不應存在之干擾因子							
		$(A1.1)$	$(A1.2)$	$(A1.3)$	$(A2)$	$(A3)$	$(A4)$	$(A5.1)$	$(A5.2)$	$(A6)$	U_{AY}	U_{1Y}	U_{2Y}	U_{A1}	U_{A2}	U_{12}	L_1	L_2
ϕ_1	$\phi_1(a', m_1', m_2')$																	
ϕ_2	$\phi_2(a', m_1', e_2', m_1')$																	
ϕ_3	$\phi_3(a', m_1', e_2', e_1')$																	
ϕ_4	$\phi_4(a', e_1', m_2')$																	
ϕ_5	$\phi_5(a', e_1', e_2', m_1')$																	
ϕ_6	$\phi_6(a', e_1', e_2', e_1')$																	

	因果參數	辨識假設									不應存在之干擾因子							
		$(A1.1)$	$(A1.2)$	$(A1.3)$	$(A2)$	$(A3)$	$(A4)$	$(A5.1)$	$(A5.2)$	$(A6)$	U_{AY}	U_{1Y}	U_{2Y}	U_{A1}	U_{A2}	U_{12}	L_1	L_2
ϕ_1	$\phi_7(A, m_1', m_2')$																	
ϕ_2	$\phi_8(A, m_1', e_2', m_1')$																	
	$\phi_9(a', m_1', A, m_1')$																	
	$\phi_{10}(A, m_1', A, m_1')$																	
ϕ_3	$\phi_{11}(a', m_1', e_2', M_1)$																	
	$\phi_{12}(A, m_1', e_2', M_1)$																	
	$\phi_{13}(a', m_1', A, M_1)$																	
	$\phi_{14}(A, m_1', A, M_1)$																	
	$\phi_{15}(A, m_1', e_2', e_1')$																	
	$\phi_{16}(a', m_1', A, e_1')$																	
	$\phi_{17}(A, m_1', A, e_1')$																	

分類	因果參數	反事實模型定義	辨識結果	可以省略的辨識假設
ϕ_4	$\phi_{18}(A,M_1,m_2')$	$E[Y(A,M_1,m_2')]$	$\Sigma_{a,m_1}E[Y\|A=a,M_1=m_1,M_2=m_2']P(M_1=m_1\|A=a)P(A=a)$	$(A1.1)$、$(A1.2)$、$(A2)$、$(A4)$
	$\phi_{19}(a',M_1,m_2')$	$E[Y(a',M_1,m_2')]$	$\Sigma_{a,m_1}E[Y\|A=a',M_1=m_1,M_2=m_2']P(M_1=m_1\|A=a)P(A=a)$	$(A1.2)$、$(A2)$、$(A4)$
	$\phi_{20}(A,e_1',m_2')$	$E[Y(A,M_1(e_1'),m_2')]$	$\Sigma_{a,m_1}E[Y\|A=a,M_1=m_1,M_2=m_2']P(M_1=m_1\|A=e_1')P(A=a)$	$(A1.1)$
ϕ_5	$\phi_{21}(a',M_1,e_2',m_1')$	$E\big[Y\big(a',M_1,M_2(e_2',m_1')\big)\big]$	$\Sigma_{a,m_1,m_2}E[Y\|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2\|A=e_2',M_1=m_1')$ $P(M_1=m_1\|A=a)P(A=a)$	$(A4)$
	$\phi_{22}(A,M_1,e_2',m_1')$	$E\big[Y\big(A,M_1,M_2(e_2',m_1')\big)\big]$	$\Sigma_{a,m_1,m_2}E[Y\|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2\|A=e_2',M_1=m_1')$ $P(M_1=m_1\|A=a)P(A=a)$	$(A1.1)$、$(A1.2)$、$(A2)$、$(A4)$
	$\phi_{23}(a',M_1,A,m_1')$	$E\big[Y\big(a',M_1,M_2(A,m_1')\big)\big]$	$\Sigma_{a,m_1,m_2}E[Y\|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2\|A=a,M_1=m_1')$ $P(M_1=m_1\|A=a)P(A=a)$	$(A4)$、$(A5.1)$、$(A6)$
	$\phi_{24}(A,M_1,A,m_1')$	$E\big[Y\big(A,M_1,M_2(A,m_1')\big)\big]$	$\Sigma_{a,m_1,m_2}E[Y\|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2\|A=a,M_1=m_1')$ $P(M_1=m_1\|A=a)P(A=a)$	$(A1.1)$、$(A1.2)$、$(A2)$、$(A4)$、$(A5.1)$、$(A6)$
	$\phi_{25}(A,e_1',e_2',m_1')$	$E\big[Y\big(A,M_1(e_1'),M_2(e_2',m_1')\big)\big]$	$\Sigma_{a,m_1,m_2}E[Y\|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2\|A=e_2',M_1=m_1')$ $P(M_1=m_1\|A=e_1')P(A=a)$	$(A1.1)$
	$\phi_{26}(a',e_1',A,m_1')$	$E\big[Y\big(a',M_1(e_1'),M_2(A,m_1')\big)\big]$	$\Sigma_{a,m_1,m_2}E[Y\|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2\|A=a,M_1=m_1')$ $P(M_1=m_1\|A=e_1')P(A=a)$	$(A5.1)$
	$\phi_{27}(A,e_1',A,m_1')$	$E\big[Y\big(A,M_1(e_1'),M_2(A,m_1')\big)\big]$	$\Sigma_{a,m_1,m_2}E[Y\|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2\|A=a,M_1=m_1')$ $P(M_1=m_1\|A=e_1')P(A=a)$	$(A1.1)$、$(A5.1)$
ϕ_6	$\phi_{28}(A,M_1,e_2',M_1)$	$E\big[Y\big(A,M_1,M_2(e_2',M_1)\big)\big]$	$\Sigma_{a,m_1,m_2}E[Y\|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2\|A=e_2',M_1=m_1)$ $P(M_1=m_1\|A=a)P(A=a)$	$(A1.1)$、$(A1.2)$、$(A2)$、$(A4)$
	$\phi_{29}(a',M_1,e_2',M_1)$	$E\big[Y\big(a',M_1,M_2(e_2',M_1)\big)\big]$	$\Sigma_{a,m_1,m_2}E[Y\|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2\|A=e_2',M_1=m_1)$ $P(M_1=m_1\|A=a)P(A=a)$	$(A4)$
	$\phi_{30}(a',M_1,A,M_1)$	$E\big[Y\big(a',M_1,M_2(A,M_1)\big)\big]$	$\Sigma_{a,m_1,m_2}E[Y\|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2\|A=a,M_1=m_1)$ $P(M_1=m_1\|A=a)P(A=a)$	$(A4)$、$(A5.1)$、$(A5.2)$、$(A6)$
	$\phi_{31}(A,e_1',e_2',e_1')$	$E\Big[Y\Big(A,M_1(e_1'),M_2\big(e_2',M_1(e_1')\big)\Big)$	$\Sigma_{a,m_1,m_2}E[Y\|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2\|A=e_2',M_1=m_1)$ $P(M_1=m_1\|A=e_1')P(A=a)$	$(A1.1)$
	$\phi_{32}(a',e_1',A,e_1')$	$E\Big[Y\Big(a',M_1(e_1'),M_2\big(A,M_1(e_1')\big)\Big)$	$\Sigma_{a,m_1,m_2}E[Y\|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2\|A=a,M_1=m_1)$ $P(M_1=m_1\|A=e_1')P(A=a)$	$(A5.1)$
	$\phi_{33}(A,e_1',A,e_1')$	$E\Big[Y\Big(A,M_1(e_1'),M_2\big(A,M_1(e_1')\big)\Big)$	$\Sigma_{a,m_1,m_2}E[Y\|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2\|A=a,M_1=m_1)$ $P(M_1=m_1\|A=e_1')P(A=a)$	$(A1.1)$、$(A3)$、$(A5.1)$

分類	因果參數	辨識假設									不應存在之干擾因子							
		$(A1.1)$	$(A1.2)$	$(A1.3)$	$(A2)$	$(A3)$	$(A4)$	$(A5.1)$	$(A5.2)$	$(A6)$	U_{AY}	U_{1Y}	U_{2Y}	U_{A1}	U_{A2}	U_{12}	L_1	L_2
ϕ_4	$\phi_{18}(A, M_1, m_2')$																	
	$\phi_{19}(a', M_1, m_2')$																	
	$\phi_{20}(A, e_1', m_2')$																	
ϕ_5	$\phi_{21}(a', M_1, e_2', m_1')$																	
	$\phi_{22}(A, M_1, e_2', m_1')$																	
	$\phi_{23}(a', M_1, A, m_1')$																	
	$\phi_{24}(A, M_1, A, m_1')$																	
	$\phi_{25}(A, e_1', e_2', m_1')$																	
	$\phi_{26}(a', e_1', A, m_1')$																	
	$\phi_{27}(A, e_1', A, m_1')$																	
ϕ_6	$\phi_{28}(A, M_1, e_2', M_1)$																	
	$\phi_{29}(a', M_1, e_2', M_1)$																	
	$\phi_{30}(a', M_1, A, M_1)$																	
	$\phi_{31}(A, e_1', e_2', e_1')$																	
	$\phi_{32}(a', e_1', A, e_1')$																	
	$\phi_{33}(A, e_1', A, e_1')$																	

3.3 PIE 三十三路徑生成機制

在此章節中，我們將第二章提及的二十八路徑拆解法拓展至雙有序中介效應及交互作用的 PIE 尺度，提出三十三路徑拆解法，整理成三十三條路徑命名為$PIE_{effect1}$至$PIE_{effect33}$，並且說明如何使用前一小節的三十三個因果參數ϕ_1至ϕ_{33}定義出這三十三條路徑特定效應的反事實模型。

(3.3.1) 雙有序中介因子的 PIE 三十三路徑最細部拆解效應

為便於理解雙有序中介因子的 PIE 三十三路徑生成機制，我們引入其因果圖，如圖 3.2 所示，與二十八路徑拆解法相同，機制分析需考量暴露因子A、中介因子M_1、M_2、交互作用項AM_1、AM_2、AM_1M_2、結果變量Y的組合。透過雙有序中介因子下的因果圖，可以直觀理解效應的生成機制，像是總共有七個箭頭指向結果變量Y，表示Y的成因可區分為以下七種生成方式：(1)A生成、(2)M_1生成、(3)M_2生成、(4)AM_1交互生成、(5)AM_2交互生成、(6)AM_1M_2交互生成、(7)M_1M_2交互生成。而中介因子M_1、M_2可能有不通過或是自行產生因果效應的情形，所以除了箭頭指向的效應生成機制外，需要加入「自行生成」、「永不生成」的條件。接著整理有M_2參與的機制且根據M_2的成因區分為：(1)A生成、(2)M_1生成、(3)AM_1交互生成、(4)自行生成、(5)永不生成；M_1的成因區分為：(1)A生成、(2)自行生成、(3)永不生成。

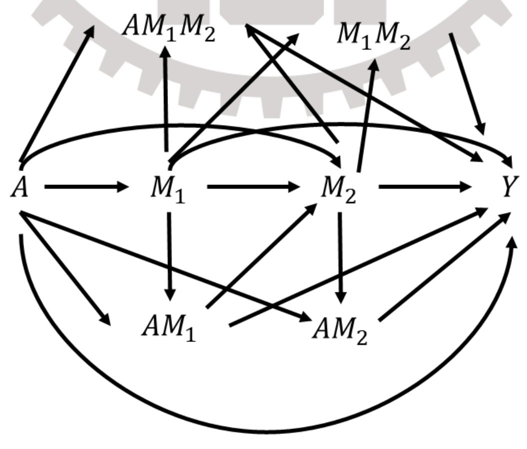


圖 3.2 雙有序中介因子下因果圖

當中三十三路徑生成機制與二十八路徑拆解法的機制組合相同，而且都可以透過因果參數表達，差別在於會多出五個路徑特定效應：$PIE_{effect29}$至$PIE_{effect33}$，分別表示中介因子M_1、M_2自行生成與交互作用直接影響結果變量Y，若是

$PIE_{effect29}$至$PIE_{effect33}$不為零，代表中介因子M_1、M_2與交互作用有直接對於Y的因果效應，另外與二十八條路徑特定效應不同的是，PIE 尺度有考慮現實世界的觀測值，所以因果參數ϕ_1至ϕ_{33}的定義，會以A表示現實世界的觀測值，下表 3.8 則整理 PIE 三十三路徑的生成機制的組合。

表 3.8 在 PIE 尺度下雙有序中介因子機制分析窮舉圖

效應	M_1生成方式	M_2生成方式	Y生成方式	路徑意義	因果參數效應定義
$PIE_{effect1}$	永不生成	永不生成	A生成	A直接影響Y，沒有通過任何中介因子	$\phi_7(A,0,0)-\phi_1(0,0,0)$
$PIE_{effect2}$	自行生成	永不生成	AM_1交互生成	A通過與M_1的交互作用影響Y，M_1自行生成	$[\phi_{20}(A,0,0)-\phi_4(0,0,0)]-[\phi_7(A,0,0)-\phi_1(0,0,0)]$
$PIE_{effect3}$	永不生成	自行生成	AM_2交互生成	A通過與M_2的交互作用影響Y，M_2自行生成	$[\phi_8(A,0,0,0)-\phi_2(0,0,0,0)]-[\phi_7(1,0,0)-\phi_1(0,0,0)]$
$PIE_{effect4}$	自行生成	自行生成	AM_1M_2交互生成	A通過與M_1和M_2交互作用(三重交互作用)影響Y，M_1和M_2是自行生成	$[\phi_{25}(A,0,0,0)-\phi_5(0,0,0,0)]-[\phi_{20}(A,0,0)-\phi_4(0,0,0)]-[\phi_8(A,0,0,0)-\phi_2(0,0,0,0)]+[\phi_7(A,0,0)-\phi_1(0,0,0)]$
$PIE_{effect5}$	自行生成	M_1生成	AM_1M_2交互生成	A通過與M_1和M_2交互作用(三重交互作用)影響Y，M_1是自行生成，M_2由M_1生成	$[\phi_{31}(A,0,0,0)-\phi_6(0,0,0,0)]-[\phi_{25}(A,0,0,0)-\phi_5(0,0,0,0)]-[\phi_{15}(A,0,0,0)-\phi_3(0,0,0,0)]+[\phi_8(A,0,0,0)-\phi_2(0,0,0,0)]$
$PIE_{effect6}$	自行生成	M_1生成	AM_2交互生成	A通過與M_2的參考交互作用影響Y，M_1自行生成，M_2由M_1生成	$[\phi_{15}(A,0,0,0)-\phi_3(0,0,0,0)]-[\phi_8(A,0,0,0)-\phi_2(0,0,0,0)]$
$PIE_{effect7}$	A生成	永不生成	M_1生成	A通過與M_1的交互作用影響Y	$\phi_{19}(0,M_1,0)-\phi_4(0,0,0)$
$PIE_{effect8}$	永不生成	A生成	M_2生成	A通過與M_2的交互作用影響Y	$\phi_9(0,0,A,0)-\phi_2(0,0,0,0)$
$PIE_{effect9}$	A生成	M_1生成	M_1M_2交互生成	A生成M_1，M_1生成M_2，M_1和M_2的交互作用影響Y	$[\phi_{29}(0,M_1,0,M_1)-\phi_6(0,0,0,0)]-[\phi_{21}(0,M_1,0,0)-\phi_5(0,0,0,0)]-[\phi_{11}(0,0,0,M_1)-\phi_3(0,0,0,0)]$
$PIE_{effect10}$	A生成	M_1生成	M_2生成	A生成M_1，M_1生成M_2，M_2再影響Y	$\phi_{11}(0,0,0,M_1)-\phi_3(0,0,0,0)$
$PIE_{effect11}$	A生成	自行生成	M_1M_2交互生成	A生成M_1，M_2自行生成，M_1和M_2的交互作用影響Y	$[\phi_{21}(0,M_1,0,0)-\phi_5(0,0,0,0)]-[\phi_{19}(0,M_1,0)-\phi_4(0,0,0)]$
$PIE_{effect12}$	自行生成	AM_1交互生成	M_2生成	M_1是自行生成，A和M_1交互作用影響M_2，M_2再影響Y	$[\phi_{16}(0,0,A,0)-\phi_3(0,0,0,0)]-[\phi_9(0,0,A,0)-\phi_2(0,0,0,0)]$
$PIE_{effect13}$	A生成	AM_1交互生成	M_2生成	A生成M_1，A和M_1交互作用影響M_2，M_2再影響Y	$[\phi_{19}(0,0,A,M_1)-\phi_{12}(0,0,0,M_1)]-[\phi_{26}(0,0,A,0)-\phi_3(0,0,0,0)]$
$PIE_{effect14}$	A生成	A生成	M_1M_2交互生成	A同時生成M_1和M_2，M_1和M_2的交互作用生成Y	$[\phi_{20}(0,M_1,A,0)-\phi_{30}(0,0,0,0)]-[\phi_{11}(0,M_1,0,0)-\phi_5(0,0,0,0)]$

$PIE_{effect15}$	自行生成	AM_1交互生成	M_1M_2交互生成	M_1是自行生成，A和M_1交互作用影響M_2，M_1和M_2的交互作用影響Y	$[\phi_{32}(0,0,A,0)-\phi_6(0,0,0,0,0)]$ $-[\phi_{26}(0,0,A,0)-\phi_5(0,0,0,0,0)]$ $-[\phi_{16}(0,0,A,0)-\phi_3(0,0,0,0,0)]$ $+[\phi_9(0,0,A,0)-\phi_2(0,0,0,0,0)]$
$PIE_{effect16}$	自行生成	A生成	M_1M_2交互生成	M_1是自行生成，A生成M_2，M_1和M_2的交互作用影響Y	$[\phi_{26}(0,0,A,0)-\phi_5(0,0,0,0,0)]$ $-[\phi_9(0,0,A,0)-\phi_2(0,0,0,0,0)]$
$PIE_{effect17}$	A生成	AM_1交互生成	M_1M_2交互生成	A生成M_1，A和M_1交互作用影響M_2，M_1和M_2的交互作用影響Y	$\begin{Bmatrix}[\phi_{21}(0,M_1,A,M_1)-\phi_{17}(0,M_1,0,M_1)]\\ -[\phi_{20}(0,M_1,A,0)-\phi_{30}(0,0,A,0)]\\ -[\phi_{19}(0,0,A,M_1)-\phi_{12}(0,0,0,M_1)]\end{Bmatrix}-\begin{Bmatrix}[\phi_{33}(0,0,A,0)-\phi_6(0,0,0,0,0)]\\ -[\phi_{11}(0,M_1,0,0)-\phi_5(0,0,0,0,0)]\\ -[\phi_{26}(0,0,A,0)-\phi_3(0,0,0,0,0)]\end{Bmatrix}$
$PIE_{effect18}$	A生成	永不生成	AM_1交互生成	A和M_1的交互作用影響Y(即A生成M_1，A和M_1交互作用影響Y)	$[\phi_{18}(A,M_1,0)-\phi_{19}(0,M_1,0)]$ $-[\phi_{20}(A,0,0)-\phi_4(0,0,0)]$
$PIE_{effect19}$	永不生成	A生成	AM_2交互生成	A和M_2的交互作用影響Y(即A生成M_2，A和M_2交互作用影響Y)	$[\phi_{10}(A,0,A,0)-\phi_8(A,0,0,0)]$ $-[\phi_9(0,0,A,0)-\phi_2(0,0,0,0,0)]$
$PIE_{effect20}$	A生成	M_1生成	AM_2交互生成	A生成M_1，M_1生成M_2，A和M_2交互作用影響Y	$[\phi_{12}(A,0,0,M_1)-\phi_{15}(A,0,0,0)]$ $-[\phi_{11}(0,0,0,M_1)-\phi_3(0,0,0,0,0)]$
$PIE_{effect21}$	A生成	自行生成	AM_1M_2交互生成	A生成M_1， M_2自行生成，A和M_1和M_2三重交互作用影響Y	$[\phi_{22}(A,M_1,0,0)-\phi_{25}(A,0,0,0)]$ $-[\phi_{21}(0,M_1,0,0)-\phi_5(0,0,0,0,0)]$ $-[\phi_{18}(A,M_1,0)-\phi_{19}(0,M_1,0)]$ $+[\phi_{20}(A,0,0)-\phi_4(0,0,0)]$
$PIE_{effect22}$	A生成	M_1生成	AM_1M_2交互生成	A生成M_1，M_1生成M_2，A和M_1和M_2三重交互作用影響Y	$\begin{Bmatrix}[\phi_{28}(A,M_1,0,M_1)-\phi_{31}(A,0,0,0)]\\ [\phi_{22}(A,M_1,0,0)-\phi_{25}(A,0,0,0)]\\ -[\phi_{12}(A,0,0,M_1)-\phi_{15}(A,0,0,0)]\end{Bmatrix}-\begin{Bmatrix}[\phi_{29}(0,M_1,0,M_1)-\phi_6(0,0,0,0,0)]\\ -[\phi_{21}(0,M_1,0,0)-\phi_5(0,0,0,0,0)]\\ -[\phi_{11}(0,0,0,M_1)-\phi_3(0,0,0,0,0)]\end{Bmatrix}$
$PIE_{effect23}$	A生成	AM_1交互生成	AM_2交互生成	A生成M_1，A和M_1交互作用影響M_2，A和M_2交互作用影響Y	$\{[\phi_{14}(A,0,A,M_1)-\phi_{17}(A,0,A,0)]$ $-[\phi_{12}(A,0,0,M_1)-\phi_{15}(A,0,0,0)]\}$ $-\{[\phi_{13}(0,0,A,M_1)-\phi_{16}(0,0,A,0)]$ $-[\phi_{11}(0,0,0,M_1)-\phi_3(0,0,0,0,0)]\}$
$PIE_{effect24}$	自行生成	AM_1交互生成	AM_2交互生成	M_1是自行生成，A和M_1交互作用影響M_2，A和M_2的交互作用影響Y	$\begin{Bmatrix}[\phi_{17}(A,0,A,0)-\phi_{15}(A,0,0,0)]\\ -[\phi_{10}(A,0,A,0)-\phi_8(A,0,0,0)]\end{Bmatrix}-\begin{Bmatrix}[\phi_{16}(0,0,A,0)-\phi_3(0,0,0,0,0)]\\ -[\phi_9(0,0,A,0)-\phi_2(0,0,0,0,0)]\end{Bmatrix}$
$PIE_{effect25}$	A生成	A生成	AM_1M_2交互生成	A同時生成M_1和M_2，A和M_1和M_2三重交互作用影響Y	$\begin{Bmatrix}[\phi_{24}(A,M_1,A,0)-\phi_{27}(A,0,A,0)]\\ -[\phi_{22}(A,M_1,0,0)-\phi_{25}(A,0,0,0)]\end{Bmatrix}-\begin{Bmatrix}[\phi_{23}(0,M_1,A,0)-\phi_{26}(0,0,A,0)]\\ -[\phi_{21}(0,M_1,0,0)-\phi_5(0,0,0,0,0)]\end{Bmatrix}$

$PIE_{effect26}$	A生成	AM_1交互生成	AM_1M_2交互生成	A生成M_1，A和M_1交互作用影響M_2，A和M_1和M_2三重交互作用影響Y	$\left\{\begin{matrix}[E(Y)-\phi_{28}(A,M_1,0,M_1)] \\ -[\phi_{24}(A,M_1,A,0)-\phi_{22}(A,M_1,0,0)] \\ -[\phi_{33}(A,0,A,0)-\phi_{31}(A,0,0,0)] \\ +[\phi_{27}(A,0,A,0)-\phi_{25}(A,0,0,0)] \\ -[\phi_{14}(A,0,A,M_1)-\phi_{17}(A,0,A,0)] \\ +[\phi_{12}(A,0,0,M_1)-\phi_{15}(A,0,0,0)]\end{matrix}\right\}$ $-\left\{\begin{matrix}[\phi_{30}(0,M_1,A,M_1)-\phi_{29}(0,M_1,0,M_1)] \\ -[\phi_{23}(0,M_1,A,0)-\phi_{21}(0,M_1,0,0)] \\ -[\phi_{32}(0,0,A,0)-\phi_6(0,0,0,0)] \\ +[\phi_{26}(0,0,A,0)-\phi_5(0,0,0,0)] \\ -[\phi_{13}(0,0,A,M_1)-\phi_{16}(0,0,A,0)] \\ +[\phi_{11}(0,0,0,M_1)-\phi_3(0,0,0,0)]\end{matrix}\right\}$
$PIE_{effect27}$	自行生成	AM_1交互生成	AM_1M_2交互生成	M_1是自行生成，A和M_1交互作用影響M_2，A和M_1和M_2三重交互作用影響Y	$\left\{\begin{matrix}[\phi_{33}(A,0,A,0)-\phi_{31}(A,0,0,0)] \\ -[\phi_{27}(A,0,A,0)-\phi_{25}(A,0,0,0)] \\ -[\phi_{17}(A,0,A,0)-\phi_{15}(A,0,0,0)] \\ +[\phi_{10}(A,0,A,0)-\phi_8(A,0,0,0)]\end{matrix}\right\}$ $-\left\{\begin{matrix}[\phi_{32}(0,0,A,0)-\phi_6(0,0,0,0)] \\ -[\phi_{26}(0,0,A,0)-\phi_5(0,0,0,0)] \\ -[\phi_{16}(0,0,A,0)-\phi_3(0,0,0,0)] \\ +[\phi_9(0,0,A,0)-\phi_2(0,0,0,0)]\end{matrix}\right\}$
$PIE_{effect28}$	自行生成	A生成	AM_1M_2交互生成	M_1是自行生成，A影響M_2，A和M_1和M_2三重交互作用影響Y	$\{[\phi_{27}(A,0,A,0)-\phi_{26}(0,0,A,0)]$ $-[\phi_{10}(A,0,A,0)-\phi_8(A,0,0,0)]\}$ $-\{[\phi_{25}(A,0,0,0)-\phi_5(0,0,0,0)]$ $-[\phi_9(0,0,A,0)-\phi_2(0,0,0,0)]\}$
$PIE_{effect29}$	自行生成	M_1生成	M_1M_2交互生成	M_1是自行生成，M_1影響M_2，M_1和M_2交互作用影響Y	$[\phi_6(0,0,0,0)-\phi_5(0,0,0,0)]$ $-[\phi_3(0,0,0,0)-\phi_2(0,0,0,0)]$
$PIE_{effect30}$	自行生成	自行生成	M_1M_2交互生成	M_1、M_2是自行生成，M_1和M_2交互作用影響Y	$[\phi_5(0,0,0,0)-\phi_2(0,0,0,0)]$ $-[\phi_4(0,0,0)-\phi_1(0,0,0)]$
$PIE_{effect31}$	自行生成	M_1生成	M_2生成	M_1是自行生成，M_1影響M_2，M_2再影響Y	$[\phi_3(0,0,0,0)-\phi_2(0,0,0,0)]$
$PIE_{effect32}$	自行生成	永不生成	M_1生成	M_1是自行生成，M_1影響Y	$[\phi_4(0,0,0)-\phi_1(0,0,0)]$
$PIE_{effect33}$	永不生成	自行生成	M_2生成	M_2是自行生成，M_2影響Y	$[\phi_2(0,0,0,0)-\phi_1(0,0,0)]$

(3.3.2) 三十三路徑拆解法反事實模型、因果圖、辨識假設之彙整

表 3.8 所呈現的效應定義主要是以巢狀反事實的框架進行表達，對此我們以更為直觀的定義，幫助研究者判讀 PIE 尺度下三十三條路徑特定效應的中介效應與交互作用的因果意義，主要表示成$Y(a,m_1,m_2)$、$M_1(e_1)$、$M_2(e_2,m_1')$的反事實模型，便於我們解釋效應的作用機制，不過其定義較為簡化，僅適用於暴露因子A、中介因子M_1、M_2為二元變量的情形。舉例而言：$PIE_{effect2}$的路徑意義是「A通過與M_1的交互作用影響Y，M_1自行生成。」，因果參數的定義為：

$$E[Y(0,M_1,0)-Y(0,M_1(0),0)]-E[Y(A,0,0)-Y(0,0,0)]$$

$$\equiv [\phi_{20}(A,0,0) - \phi_4(0,0,0)] - [\phi_7(A,0,0) - \phi_1(0,0,0)]$$

可以進一步拆解為以下的效應定義：

$$E\{[Y(1,1,0) - Y(1,0,0) - Y(0,1,0) + Y(0,0,0)] * A * M_1(0)\}$$

當中的$[Y(1,1,0) - Y(1,0,0) - Y(0,1,0) + Y(0,0,0)]$可以較為直觀的解讀成暴露因子$A$、中介因子$M_1$對$Y$交互作用，$M_1(0)$則表示中介因子$M_1$自行生成的效應，如同第二章(2.2.2)使用較為直觀的定義，可以幫助研究者直觀的繪製出各個效應的因果圖，以下的表 3.9 則是三十三個族群介入效應的因果圖整理、以及表 3.10 族群介入效應的反事實模型定義、因果圖、辨識假設之彙整。

表 3.9 三十三個族群介入效應的因果圖整理

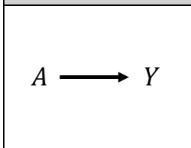
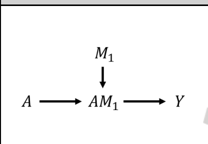
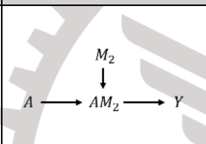
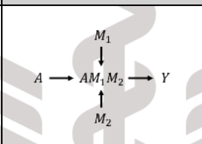
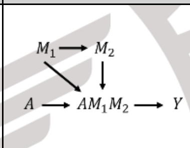
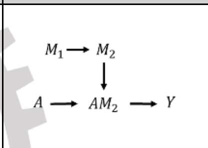
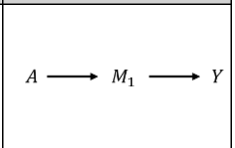
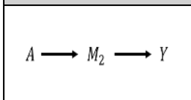
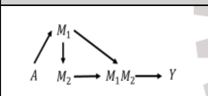
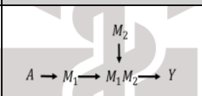
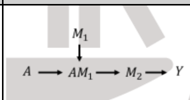
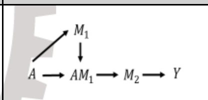
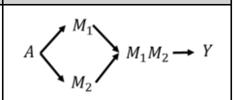
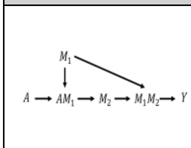
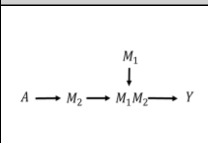
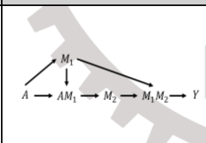
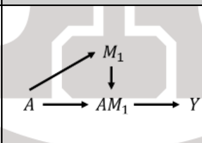
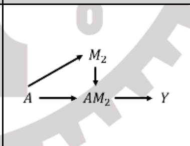
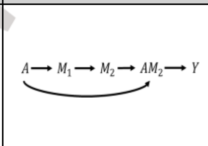
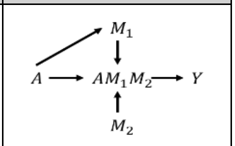
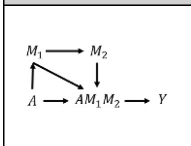
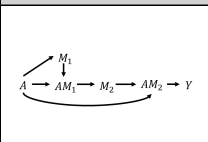
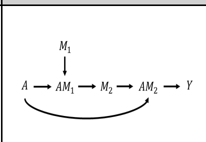
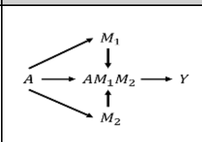
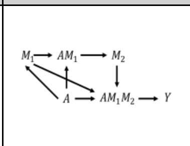
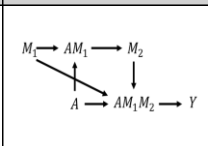
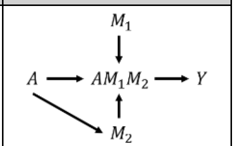
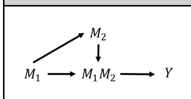
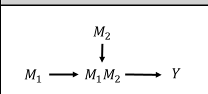
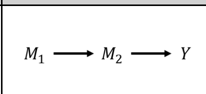
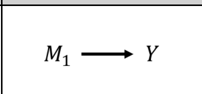
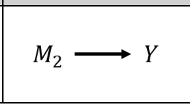
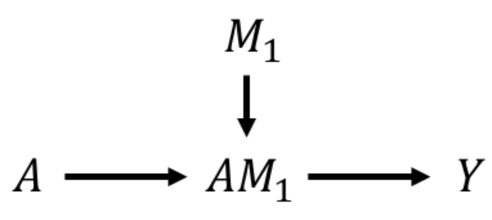
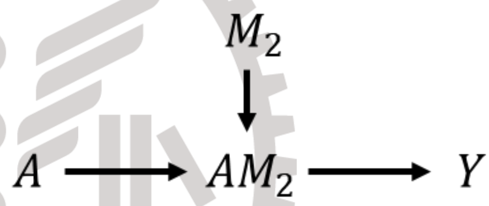
$PIE_{effect1}$	$PIE_{effect2}$	$PIE_{effect3}$	$PIE_{effect4}$	$PIE_{effect5}$	$PIE_{effect6}$	$PIE_{effect7}$
						
$PIE_{effect8}$	$PIE_{effect9}$	$PIE_{effect10}$	$PIE_{effect11}$	$PIE_{effect12}$	$PIE_{effect13}$	$PIE_{effect14}$
						
$PIE_{effect15}$	$PIE_{effect16}$	$PIE_{effect17}$	$PIE_{effect18}$	$PIE_{effect19}$	$PIE_{effect20}$	$PIE_{effect21}$
						
$PIE_{effect22}$	$PIE_{effect23}$	$PIE_{effect24}$	$PIE_{effect25}$	$PIE_{effect26}$	$PIE_{effect27}$	$PIE_{effect28}$
						
$PIE_{effect29}$	$PIE_{effect30}$	$PIE_{effect31}$	$PIE_{effect32}$	$PIE_{effect33}$		
						

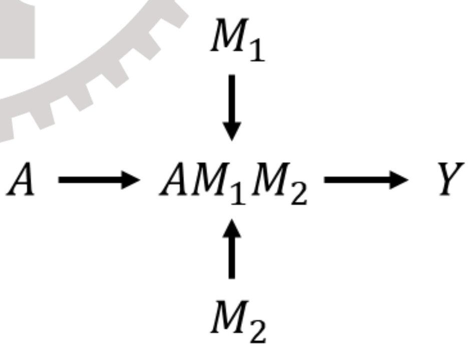
表 3.10 族群介入效應的反事實模型定義、因果圖、辨識假設之彙整

<table>
<tr><th colspan="2">$PIE_{effect1}$</th></tr>
<tr><td colspan="2">$Definition$</td></tr>
<tr><td colspan="2">$E[Y(A,0,0) - Y(0,0,0)]$
$= \phi_7(A,0,0) - \phi_1(0,0,0)$</td></tr>
<tr><td colspan="2">$Identification$</td></tr>
<tr><td colspan="2">$\phi_1 = E[Y(a',m_1',m_2')] = E[Y|A = a', M_1 = m_1', M_2 = m_2']$
$\phi_7 = E[Y(A,m_1',m_2')] = \Sigma_a E[Y|A = a, M_1 = m_1', M_2 = m_2']P(A = a)$</td></tr>
<tr><td>$Required\ Assumptions$</td><td>DAG</td></tr>
<tr><td>$(A1)\ Y(a,m_1,m_2) \perp \{A, M_1, M_2\}$
• $(A1.1)\ Y(a,m_1,m_2) \perp A$
• $(A1.2)\ Y(a,m_1,m_2) \perp M_1|A$
• $(A1.3)\ Y(a,m_1,m_2) \perp M_2|A, M_1$</td><td></td></tr>
</table>

<table>
<tr><th colspan="2">$PIE_{effect2}$</th></tr>
<tr><td colspan="2">$Definition$</td></tr>
<tr><td colspan="2">$E\{[Y(1,1,0) - Y(1,0,0) - Y(0,1,0) + Y(0,0,0)] * A * M_1(0)\}$
$= E[Y(A, M_1(0), 0) - Y(0, M_1(0), 0)] - E[Y(A,0,0) - Y(0,0,0)]$
$= [\phi_{20}(A,0,0) - \phi_4(0,0,0)] - [\phi_7(A,0,0) - \phi_1(0,0,0)]$</td></tr>
<tr><td colspan="2">$Identification$</td></tr>
<tr><td colspan="2">$\phi_1 = E[Y(a',m_1',m_2')] = E[Y|A = a', M_1 = m_1', M_2 = m_2']$
$\phi_7 = E[Y(A,m_1',m_2')] = \Sigma_a E[Y|A = a, M_1 = m_1', M_2 = m_2']P(A = a)$
$\phi_4 = E[Y(a', M_1(e_1'), m_2')] = \Sigma_{m_1} E[Y|A = a', M_1 = m_1, M_2 = m_2']P(M_1 = m_1|A = e_1')$
$\phi_{20} = E[Y(A, M_1(e_1'), m_2')] = \Sigma_{a,m_1} E[Y|A = a, M_1 = m_1, M_2 = m_2']P(M_1 = m_1|A = e_1')P(A = a)$</td></tr>
<tr><td>$Required\ Assumptions$</td><td>DAG</td></tr>
<tr><td>• $(A1.2)\ Y(a,m_1,m_2) \perp M_1|A$
• $(A1.3)\ Y(a,m_1,m_2) \perp M_2|A, M_1$
$(A2)\ Y(a,m_1,m_2) \perp M_1(e_1)$
$(A4)\ M_1(e_1) \perp A$</td><td></td></tr>
</table>

<table>
<tr><th colspan="2">$PIE_{effect3}$</th></tr>
<tr><th colspan="2">$Definition$</th></tr>
<tr><td colspan="2">$E\{[Y(1,0,1) - Y(1,0,0) - Y(0,0,1) + Y(0,0,0)] * A * M_2(0,0)\}$
$= E[Y(A, 0, M_2(0,0)) - Y(0,0, M_2(0,0))] - E[Y(A,0,0) - Y(0,0,0)]$
$= [\phi_8(A,0,0,0) - \phi_2(0,0,0,0)] - [\phi_7(A,0,0) - \phi_1(0,0,0)]$</td></tr>
<tr><th colspan="2">$Identification$</th></tr>
<tr><td colspan="2">$\phi_1 = E[Y(a', m_1', m_2')] = E[Y|A = a', M_1 = m_1', M_2 = m_2']$
$\phi_2 = E[Y(a', m_1', M_2(e_2', m_1'))] = \Sigma_{a,m_1,m_2} E[Y|A = a', M_1 = m_1', M_2 = m_2] P(M_2 = m_2|A = e_2', M_1 = m_1')$
$\phi_7 = E[Y(A, m_1', m_2')] = \Sigma_{a,m_1,m_2} E[Y|A = a, M_1 = m_1', M_2 = m_2'] P(A = a)$
$\phi_8 = E[Y(A, m_1', M_2(e_2', m_1'))]$
$= \Sigma_{a,m_2} E[Y|A = a, M_1 = m_1', M_2 = m_2] P(M_2 = m_2|A = e_2', M_1 = m_1') P(A = a)$</td></tr>
<tr><th>$Required\ Assumptions$</th><th>DAG</th></tr>
<tr><td>• $(A1.2)\ Y(a, m_1, m_2) \perp M_1|A$
• $(A1.3)\ Y(a, m_1, m_2) \perp M_2|A, M_1$

$(A3)\ Y(a, m_1, m_2) \perp M_2(e_2, m_1')$
$(A5)\ M_2(e_2, m_1') \perp \{A, M_1\}$
• $(A5.1)\ M_2(e_2, m_1') \perp A$
• $(A5.2)\ M_2(e_2, m_1') \perp M_1|A$</td><td></td></tr>
</table>

<table>
<tr><th colspan="2">$PIE_{effect4}$</th></tr>
<tr><th colspan="2">$Definition$</th></tr>
<tr><td colspan="2">$$E\{[Y(1,1,1)-Y(1,1,0)-Y(1,0,1)+Y(1,0,0)-Y(0,1,1)+Y(0,1,0)+Y(0,0,1)-Y(0,0,0)]*A*M_1(0)*M_2(0,0)\}$$
$$=E\big[Y\big(A.M_1(0),M_2(0,0)\big)-Y\big(0,M_1(0),M_2(0,0)\big)\big]-E[Y(A,M_1(0),0)-Y(0,M_1(0),0)]-E\big[Y\big(A,0,M_2(0,0)\big)-Y\big(0,0,M_2(0,0)\big)\big]+E[Y(A,0,0)-Y(0,0,0)]$$
$$=[\phi_{25}(A,0,0,0)-\phi_5(0,0,0,0)]-[\phi_{20}(A,0,0)-\phi_4(0,0,0)]-[\phi_8(A,0,0,0)-\phi_2(0,0,0,0)]+[\phi_7(A,0,0)-\phi_1(0,0,0)]$$</td></tr>
<tr><th colspan="2">$Identification$</th></tr>
<tr><td colspan="2">$\phi_1=E[Y(a',m_1',m_2')]=E[Y|A=a',M_1=m_1',M_2=m_2']$
$\phi_2=E\big[Y\big(a',m_1',M_2(e_2',m_1')\big)\big]=\Sigma_{m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')$
$\phi_4=E[Y(a',M_1(e_1'),m_2')]=\Sigma_{m_1}E[Y|A=a',M_1=m_1,M_2=m_2']P(M_1=m_1|A=e_1')$
$\phi_5=E\big[Y\big(a',M_1(e_1'),M_2(e_2',m_1')\big)\big]$
$=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(M_1=m_1|A=e_1')$
$\phi_7=E[Y(A,m_1',m_2')]=\Sigma_aE[Y|A=a,M_1=m_1',M_2=m_2']P(A=a)$
$\phi_{10}=E\big[Y\big(A,m_1',M_2(e_2',m_1')\big)\big]$
$=\Sigma_{a,m_2}E[Y|A=a,M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(A=a)$
$\phi_{20}=E[Y(A,M_1(e_1'),m_2')]=\Sigma_{a,m_1}E[Y|A=a,M_1=m_1,M_2=m_2']P(M_1=m_1|A=e_1')P(A=a)$
$\phi_{25}=E\big[Y\big(A,M_1(e_1'),M_2(e_2',m_1')\big)\big]$
$=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(M_1=m_1|A=e_1')P(A=a)$</td></tr>
<tr><th>$Required\ Assumptions$</th><th>DAG</th></tr>
<tr><td>$(A1)\ Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$
• $(A1.1)\ Y(a,m_1,m_2)\perp A$
• $(A1.2)\ Y(a,m_1,m_2)\perp M_1|A$
• $(A1.3)\ Y(a,m_1,m_2)\perp M_2|A,M_1$
$(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$
$(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$
$(A4)\ M_1(e_1)\perp A$
$(A5)\ M_2(e_2,m_1')\perp\{A,M_1\}$
• $(A5.1)\ M_2(e_2,m_1)\perp A$
• $(A5.2)\ M_2(e_2,m_1)\perp M_1|A$
$(A6)\ M_2(e_2,m_1')\perp M_1(e_1)$</td><td>
</td></tr>
</table>

$PIE_{effect5}$

Definition

$$E\{[Y(1,1,1) - Y(1,1,0) - Y(0,1,1) + Y(0,1,0) - Y(1,0,1) + Y(1,0,0) + Y(0,0,1) - Y(0,0,0)] * A * M_1(0) * [M_2(0,1) - M_2(0,0)]\}$$

$$= E\left[Y\left(A, M_1(0), M_2\left(0, M_1(0)\right)\right) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right] - E\left[Y\left(A, M_1(0), M_2(0,0)\right) - Y\left(0, M_1(0), M_2(0,0)\right)\right] - E\left[Y\left(A, 0, M_2\left(0, M_1(0)\right)\right) - Y\left(0,0, M_2\left(0, M_1(0)\right)\right)\right] + E\left[Y\left(A, 0, M_2(0,0)\right) - Y\left(0,0, M_2(0,0)\right)\right]$$

$$= [\phi_{31}(A, 0,0,0) - \phi_6(0,0,0,0)] - [\phi_{25}(A, 0,0,0) - \phi_5(0,0,0,0)] - [\phi_{15}(A, 0,0,0) - \phi_3(0,0,0,0)] + [\phi_8(A, 0,0,0) - \phi_2(0,0,0,0)]$$

Identification

$\phi_2 = E\left[Y\left(a', m_1', M_2(e_2', m_1')\right)\right] = \Sigma_{a,m_1,m_2} E[Y|A = a', M_1 = m_1', M_2 = m_2] P(M_2 = m_2|A = e_2', M_1 = m_1')$

$\phi_3 = E\left[Y\left(a', m_1', M_2\left(e_2', M_1(e_1')\right)\right)\right] = \Sigma_{a,m_1,m_2} E[Y|A = a', M_1 = m_1', M_2 = m_2] P(M_2 = m_2|A = e_2', M_1 = m_1) P(M_1 = m_1|A = e_1')$

$\phi_5 = E\left[Y\left(a', M_1(e_1'), M_2(e_2', m_1')\right)\right] = \Sigma_{a,m_1,m_2} E[Y|A = a', M_1 = m_1, M_2 = m_2] P(M_2 = m_2|A = e_2', M_1 = m_1') P(M_1 = m_1|A = e_1')$

$\phi_6 = E\left[Y\left(a', M_1(e_1'), M_2\left(e_2', M_1(e_1')\right)\right)\right] = \Sigma_{a,m_1,m_2} E[Y|A = a', M_1 = m_1, M_2 = m_2] P(M_2 = m_2|A = e_2', M_1 = m_1) P(M_1 = m_1|A = e_1')$

$\phi_8 = E\left[Y\left(A, m_1', M_2(e_2', m_1')\right)\right] = \Sigma_{a,m_1,m_2} E[Y|A = a, M_1 = m_1', M_2 = m_2] P(M_2 = m_2|A = e_2', M_1 = m_1') P(A = a)$

$\phi_{15} = E\left[Y\left(A, m_1', M_2\left(e_2', M_1(e_1')\right)\right)\right]$

$= \Sigma_{a,m_1,m_2} E[Y|A = a, M_1 = m_1', M_2 = m_2] P(M_2 = m_2|A = e_2', M_1 = m_1) P(M_1 = m_1|A = e_1') P(A = a)$

$\phi_{25} = E\left[Y\left(A, m_1', M_2\left(e_2', M_1(e_1')\right)\right)\right]$

$= \Sigma_{a,m_1,m_2} E[Y|A = a, M_1 = m_1', M_2 = m_2] P(M_2 = m_2|A = e_2', M_1 = m_1) P(M_1 = m_1|A = e_1') P(A = a)$

$\phi_{31} = E\left[Y\left(A, M_1(e_1'), M_2\left(e_2', M_1(e_1')\right)\right)\right]$

$= \Sigma_{a,m_1,m_2} E[Y|A = a, M_1 = m_1, M_2 = m_2] P(M_2 = m_2|A = e_2', M_1 = m_1) P(M_1 = m_1|A = e_1') P(A = a)$

Required Assumptions

$(A1)\ Y(a, m_1, m_2) \perp \{A, M_1, M_2\}$

- $(A1.1)\ Y(a, m_1, m_2) \perp A$
- $(A1.2)\ Y(a, m_1, m_2) \perp M_1|A$
- $(A1.3)\ Y(a, m_1, m_2) \perp M_2|A, M_1$

$(A2)\ Y(a, m_1, m_2) \perp M_1(e_1)$

$(A3)\ Y(a, m_1, m_2) \perp M_2(e_2, m_1')$

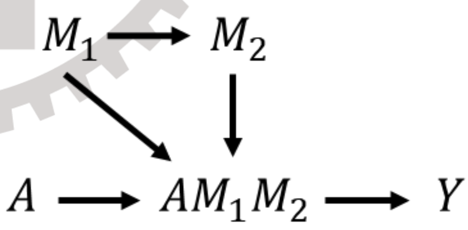
$(A4)\ M_1(e_1) \perp A$

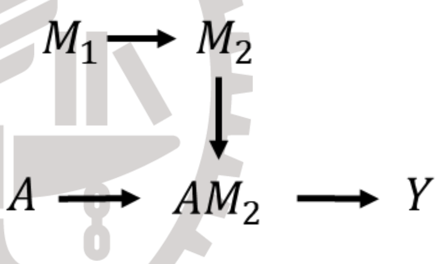
$(A5)\ M_2(e_2, m_1') \perp \{A, M_1\}$

- $(A5.1)\ M_2(e_2, m_1') \perp A$
- $(A5.2)\ M_2(e_2, m_1') \perp M_1|A$

$(A6)\ M_2(e_2, m_1') \perp M_1(e_1)$

DAG



<table>
<tr><th colspan="2">$PIE_{effect6}$</th></tr>
<tr><th colspan="2">$Definition$</th></tr>
<tr><td colspan="2">$E\left[Y\left(A,0,M_2\left(0,M_1(0)\right)\right)-Y\left(0,0,M_2\left(0,M_1(0)\right)\right)\right]-E\left[Y\left(A,0,M_2(0,0)\right)-Y\left(0,0,M_2(0,0)\right)\right]$
$=E\{[Y(1,0,1)-Y(1,0,0)-Y(0,0,1)+Y(0,0,0)]*A*M_1(0)*[M_2(0,1)-M_2(0,0)]\}$
$=[\phi_{15}(A,0,0,0)-\phi_3(0,0,0,0)]-[\phi_8(A,0,0,0)-\phi_2(0,0,0,0)]$</td></tr>
<tr><th colspan="2">$Identification$</th></tr>
<tr><td colspan="2">$\phi_2=E\left[Y\left(a',m_1',M_2(e_2',m_1')\right)\right]$
$=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')$
$\phi_3=E\left[Y\left(a',m_1',M_2\left(e_2',M_1(e_1')\right)\right)\right]$
$=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=e_1')$
$\phi_8=E\left[Y\left(A,m_1',M_2(e_2',m_1')\right)\right]$
$=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(A=a)$
$\phi_{15}=E\left[Y\left(A,m_1',M_2\left(e_2',M_1(e_1')\right)\right)\right]$
$=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=e_1')P(A=a)$</td></tr>
<tr><th>$Required\ Assumptions$</th><th>DAG</th></tr>
<tr><td>$(A1)\ Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$
● $(A1.1)\ Y(a,m_1,m_2)\perp A$
● $(A1.2)\ Y(a,m_1,m_2)\perp M_1|A$
● $(A1.3)\ Y(a,m_1,m_2)\perp M_2|A,M_1$
$(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$
$(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$
$(A4)\ M_1(e_1)\perp A$
$(A5)\ M_2(e_2,m_1')\perp\{A,M_1\}$
● $(A5.1)\ M_2(e_2,m_1')\perp A$
● $(A5.2)\ M_2(e_2,m_1')\perp M_1|A$
$(A6)\ M_2(e_2,m_1')\perp M_1(e_1)$</td><td></td></tr>
</table>

$PIE_{effect7}$	
Definition	
$E\{[Y(0,1,0)-Y(0,0,0)][M_1-M_1(0)]\}$ $=E[Y(0,M_1,0)-Y(0,M_1(0),0)]$ $=\phi_{19}(0,M_1,0)-\phi_4(0,0,0)$	
Identification	
$\phi_4=E[Y(a',M_1(e_1'),m_2')]$ $=\Sigma_{m_1}E[Y\|A=a',M_1=m_1,M_2=m_2']P(M_1=m_1\|A=e_1')$ $\phi_{19}=E[Y(a',M_1,m_2')]$ $=\Sigma_{a,m_1}E[Y\|A=a',M_1=m_1,M_2=m_2']P(M_1=m_1\|A=a)P(A=a)$	
Required Assumptions	*DAG*
$(A1)\ Y(a,m_1,m_2)\perp\{A\ ,M_1,M_2\}$ • $(A1.1)\ Y(a,m_1,m_2)\perp A$ • $(A1.2)\ Y(a,m_1,m_2)\perp M_1\|A$ • $(A1.3)\ Y(a,m_1,m_2)\perp M_2\|A,M_1$ $(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$ $(A4)\ M_1(e_1)\perp A$	

<table>
<tr><th colspan="2">$PIE_{effect8}$</th></tr>
<tr><th colspan="2">$Definition$</th></tr>
<tr><td colspan="2">$E\{[Y(0,0,1) - Y(0,0,0)][M_2(1,0) - M_2(0,0)] * A\}$
$= E\big[Y\big(0,0, M_2(A,0)\big) - Y\big(0,0, M_2(0,0)\big)\big]$
$= \phi_9(0,0,A,0) - \phi_2(0,0,0,0)$</td></tr>
<tr><th colspan="2">$Identification$</th></tr>
<tr><td colspan="2">$\phi_2 = E\big[Y\big(a', m_1', M_2(e_2', m_1')\big)\big]$
$= \Sigma_{m_2} E[Y \mid A = a', M_1 = m_1', M_2 = m_2] P(M_2 = m_2 \mid A = e_2', M_1 = m_1')$
$\phi_9 = E\big[Y\big(a', m_1', M_2(A, m_1')\big)\big]$
$= \Sigma_{a,m_2} E[Y \mid A = a', M_1 = m_1', M_2 = m_2] P(M_2 = m_2 \mid A = a, M_1 = m_1') P(A = a)$</td></tr>
<tr><th>$Required\ Assumptions$</th><th>DAG</th></tr>
<tr><td>$(A1)\ Y(a, m_1, m_2) \perp \{A\ , M_1, M_2\}$
• $(A1.1)\ Y(a, m_1, m_2) \perp A$
• $(A1.2)\ Y(a, m_1, m_2) \perp M_1 \mid A$
• $(A1.3)\ Y(a, m_1, m_2) \perp M_2 \mid A, M_1$
$(A3)\ Y(a, m_1, m_2) \perp M_2(e_2, m_1')$
$(A5)\ M_2(e_2, m_1') \perp \{A, M_1\}$
• $(A5.1)\ M_2(e_2, m_1') \perp A$
• $(A5.2)\ M_2(e_2, m_1') \perp M_1 \mid A$</td><td></td></tr>
</table>

<table>
<tr><th colspan="2">$PIE_{effect9}$</th></tr>
<tr><th colspan="2">$Definition$</th></tr>
<tr><td colspan="2">

$E\{[Y(0,1,1) - Y(0,1,0) - Y(0,0,1) + Y(0,0,0)][M_1 - M_1(0)][M_2(0,1) - M_2(0,0)]\}$

$= E\left[Y\big(0, M_1, M_2(0, M_1)\big) - Y\left(0, M_1(0), M_2\big(0, M_1(0)\big)\right)\right]$

$\quad - E\big[Y\big(0, M_1, M_2(0,0)\big) - Y\big(0, M_1(0), M_2(0,0)\big)\big]$

$\quad - E\left[Y\big(0,0, M_2(0, M_1)\big) - Y\left(0,0, M_2\big(0, M_1(0)\big)\right)\right]$

$= [\phi_{29}(0, M_1, 0, M_1) - \phi_6(0,0,0,0)] - [\phi_{21}(0, M_1, 0,0) - \phi_5(0,0,0,0)]$

$\quad - [\phi_{11}(0,0,0, M_1) - \phi_3(0,0,0,0)]$

</td></tr>
<tr><th colspan="2">$Identification$</th></tr>
<tr><td colspan="2">

$\phi_3 = E\left[Y\left(a', m_1', M_2\big(e_2', M_1(e_1')\big)\right)\right]$

$= \Sigma_{m_1,m_2} E[Y|A = a', M_1 = m_1', M_2 = m_2] P(M_2 = m_2|A = e_2', M_1 = m_1) P(M_1 = m_1|A = e_1')$

$\phi_5 = E\big[Y\big(a', M_1(e_1'), M_2(e_2', m_1')\big)\big]$

$= \Sigma_{m_1,m_2} E[Y|A = a', M_1 = m_1, M_2 = m_2] P(M_2 = m_2|A = e_2', M_1 = m_1') P(M_1 = m_1|A = e_1')$

$\phi_6 = E\left[Y\left(a', M_1(e_1'), M_2\big(e_2', M_1(e_1')\big)\right)\right]$

$= \Sigma_{m_1,m_2} E[Y|A = a', M_1 = m_1, M_2 = m_2] P(M_2 = m_2|A = e_2', M_1 = m_1) P(M_1 = m_1|A = e_1')$

$\phi_{11} = E\big[Y\big(a', m_1', M_2(e_2', M_1)\big)\big]$

$= \Sigma_{m_1,m_2} E[Y|A = a', M_1 = m_1', M_2 = m_2] P(M_2 = m_2|A = e_2', M_1 = m_1) P(M_1 = m_1)$

$\phi_{21} = E\big[Y\big(a', M_1, M_2(e_2', m_1')\big)\big]$

$= \Sigma_{m_1,m_2} E[Y|A = a', M_1 = m_1, M_2 = m_2] P(M_2 = m_2|A = e_2', M_1 = m_1') P(M_1 = m_1)$

$\phi_{29} = E\big[Y\big(a', M_1, M_2(e_2', M_1)\big)\big]$

$= \Sigma_{m_1,m_2} E[Y|A = a', M_1 = m_1, M_2 = m_2] P(M_2 = m_2|A = e_2', M_1 = m_1) P(M_1 = m_1)$

</td></tr>
<tr><th>$Required\ Assumptions$</th><th>DAG</th></tr>
<tr><td>

$(A1)\ Y(a, m_1, m_2) \perp \{A, M_1, M_2\}$

- $(A1.1)\ Y(a, m_1, m_2) \perp A$
- $(A1.2)\ Y(a, m_1, m_2) \perp M_1|A$
- $(A1.3)\ Y(a, m_1, m_2) \perp M_2|A, M_1$

$(A2)\ Y(a, m_1, m_2) \perp M_1(e_1)$

$(A3)\ Y(a, m_1, m_2) \perp M_2(e_2, m_1')$

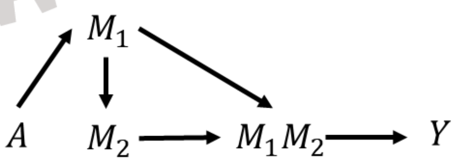
$(A4)\ M_1(e_1) \perp A$

$(A5)\ M_2(e_2, m_1') \perp \{A, M_1\}$

- $(A5.1)\ M_2(e_2, m_1') \perp A$
- $(A5.2)\ M_2(e_2, m_1') \perp M_1|A$

$(A6)\ M_2(e_2, m_1') \perp M_1(e_1)$

</td><td>



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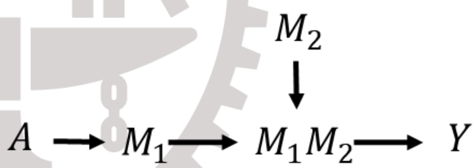
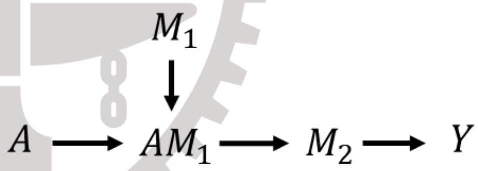
</table>

<table>
<tr><th colspan="2">$PIE_{effect10}$</th></tr>
<tr><th colspan="2">$Definition$</th></tr>
<tr><td colspan="2">$E\{[Y(0,0,1) - Y(0,0,0)][M_2(0,1) - M_2(0,0)][M_1 - M_1(0)]\}$
$= E\left[Y\big(0,0,M_2(0,M_1)\big) - Y\left(0,0,M_2\big(0,M_1(0)\big)\right)\right]$
$= \phi_{11}(0,0,0,M_1) - \phi_3(0,0,0,0)$</td></tr>
<tr><th colspan="2">$Identification$</th></tr>
<tr><td colspan="2">$\phi_3 = E\left[Y\left(a', m_1', M_2\big(e_2', M_1(e_1')\big)\right)\right]$
$= \Sigma_{m_1,m_2} E[Y \mid A = a', M_1 = m_1', M_2 = m_2] P(M_2 = m_2 \mid A = e_2', M_1 = m_1) P(M_1 = m_1 \mid A = e_1')$
$\phi_{11} = E\big[Y\big(a', m_1', M_2(e_2', M_1)\big)\big]$
$= \Sigma_{m_1,m_2} E[Y \mid A = a', M_1 = m_1', M_2 = m_2] P(M_2 = m_2 \mid A = e_2', M_1 = m_1) P(M_1 = m_1)$</td></tr>
<tr><th>$Required\ Assumptions$</th><th>DAG</th></tr>
<tr><td>$(A1)\ Y(a, m_1, m_2) \perp \{A, M_1, M_2\}$
• $(A1.1)\ Y(a, m_1, m_2) \perp A$
• $(A1.2)\ Y(a, m_1, m_2) \perp M_1 \mid A$
• $(A1.3)\ Y(a, m_1, m_2) \perp M_2 \mid A, M_1$
$(A2)\ Y(a, m_1, m_2) \perp M_1(e_1)$
$(A3)\ Y(a, m_1, m_2) \perp M_2(e_2, m_1')$
$(A4)\ M_1(e_1) \perp A$
$(A5)\ M_2(e_2, m_1') \perp \{A, M_1\}$
• $(A5.1)\ M_2(e_2, m_1') \perp A$
• $(A5.2)\ M_2(e_2, m_1') \perp M_1 \mid A$
$(A6)\ M_2(e_2, m_1') \perp M_1(e_1)$</td><td>$A \longrightarrow M_1 \longrightarrow M_2 \longrightarrow Y$</td></tr>
</table>

<table>
<tr><th colspan="2">$PIE_{effect11}$</th></tr>
<tr><th colspan="2">$Definition$</th></tr>
<tr><td colspan="2">$E\{[Y(0,1,1) - Y(0,1,0) - Y(0,0,1) + Y(0,0,0)][M_1 - M_1(0)]M_2(0,0)\}$
$= E\left[Y\left(0, M_1, M_2(0,0)\right) - Y\left(0, M_1(0), M_2(0,0)\right)\right]$
$- E[Y(0, M_1, 0) - Y(0, M_1(0), 0)]$
$= [\phi_{21}(0, M_1, 0,0) - \phi_5(0,0,0,0)] - [\phi_{19}(0, M_1, 0) - \phi_4(0,0,0)]$</td></tr>
<tr><th colspan="2">$Identification$</th></tr>
<tr><td colspan="2">$\phi_4 = E[Y(a', M_1(e_1'), m_2')]$
$= \Sigma_{m_1} E[Y|A = a', M_1 = m_1, M_2 = m_2']P(M_1 = m_1|A = e_1')$
$\phi_5 = E\left[Y\left(a', M_1(e_1'), M_2(e_2', m_1')\right)\right]$
$= \Sigma_{m_1,m_2} E[Y|A = a', M_1 = m_1, M_2 = m_2]P(M_2 = m_2|A = e_2', M_1 = m_1')P(M_1 = m_1|A = e_1')$
$\phi_{19} = E[Y(a', M_1, m_2')]$
$= \Sigma_{m_1} E[Y|A = a', M_1 = m_1, M_2 = m_2']P(M_1 = m_1)$
$\phi_{21} = E\left[Y\left(a', M_1, M_2(e_2', m_1')\right)\right]$
$= \Sigma_{m_1,m_2} E[Y|A = a', M_1 = m_1, M_2 = m_2]P(M_2 = m_2|A = e_2', M_1 = m_1')P(M_1 = m_1)$</td></tr>
<tr><th>$Required\ Assumptions$</th><th>DAG</th></tr>
<tr><td>$(A1)\ Y(a, m_1, m_2) \perp \{A, M_1, M_2\}$
• $(A1.1)\ Y(a, m_1, m_2) \perp A$
• $(A1.2)\ Y(a, m_1, m_2) \perp M_1|A$
• $(A1.3)\ Y(a, m_1, m_2) \perp M_2|A, M_1$
$(A2)\ Y(a, m_1, m_2) \perp M_1(e_1)$
$(A3)\ Y(a, m_1, m_2) \perp M_2(e_2, m_1')$
$(A4)\ M_1(e_1) \perp A$
$(A5)\ M_2(e_2, m_1') \perp \{A, M_1\}$
• $(A5.1)\ M_2(e_2, m_1') \perp A$
• $(A5.2)\ M_2(e_2, m_1') \perp M_1|A$
$(A6)\ M_2(e_2, m_1') \perp M_1(e_1)$</td><td>
</td></tr>
</table>

<table>
<tr><th colspan="2">$PIE_{effect12}$</th></tr>
<tr><td colspan="2" align="center">$Definition$</td></tr>
<tr><td colspan="2">
$E\{[Y(0,0,1) - Y(0,0,0)][M_2(1,1) - M_2(1,0) - M_2(0,1) + M_2(0,0)] * A * M_1(0)\}$
$= E\left[Y\left(0,0,M_2\left(A,M_1(0)\right)\right) - Y\left(0,0,M_2\left(0,M_1(0)\right)\right)\right]$
$- E\left[Y\left(0,0,M_2(A,0)\right) - Y\left(0,0,M_2(0,0)\right)\right]$
$= [\phi_{16}(0,0,A,0) - \phi_3(0,0,0,0)] - [\phi_9(0,0,A,0) - \phi_2(0,0,0,0)]$
</td></tr>
<tr><td colspan="2" align="center">$Identification$</td></tr>
<tr><td colspan="2">
$\phi_2 = E\left[Y\left(a', m_1', M_2(e_2', m_1')\right)\right]$
$= \Sigma_{m_2} E[Y|A = a', M_1 = m_1', M_2 = m_2] P(M_2 = m_2|A = e_2', M_1 = m_1')$
$\phi_3 = E\left[Y\left(a', m_1', M_2\left(e_2', M_1(e_1')\right)\right)\right]$
$= \Sigma_{m_1,m_2} E[Y|A = a', M_1 = m_1', M_2 = m_2] P(M_2 = m_2|A = e_2', M_1 = m_1) P(M_1 = m_1|A = e_1')$
$\phi_9 = E\left[Y\left(a', m_1', M_2(A, m_1')\right)\right]$
$= \Sigma_{a,m_2} E[Y|A = a', M_1 = m_1', M_2 = m_2] P(M_2 = m_2|A = a, M_1 = m_1') P(A = a)$
$\phi_{16} = E\left[Y\left(a', m_1', M_2\left(A, M_1(e_1')\right)\right)\right]$
$= \Sigma_{a,m_1,m_2} E[Y|A = a', M_1 = m_1', M_2 = m_2] P(M_2 = m_2|A = a, M_1 = m_1) P(M_1 = m_1|A = e_1') P(A = a)$
</td></tr>
<tr><td align="center">$Required\ Assumptions$</td><td align="center">DAG</td></tr>
<tr><td>
$(A1)\ Y(a, m_1, m_2) \perp \{A, M_1, M_2\}$
● $(A1.1)\ Y(a, m_1, m_2) \perp A$
● $(A1.2)\ Y(a, m_1, m_2) \perp M_1|A$
● $(A1.3)\ Y(a, m_1, m_2) \perp M_2|A, M_1$
$(A2)\ Y(a, m_1, m_2) \perp M_1(e_1)$
$(A3)\ Y(a, m_1, m_2) \perp M_2(e_2, m_1')$
$(A4)\ M_1(e_1) \perp A$
$(A5)\ M_2(e_2, m_1') \perp \{A, M_1\}$
● $(A5.1)\ M_2(e_2, m_1') \perp A$
● $(A5.2)\ M_2(e_2, m_1') \perp M_1|A$
$(A6)\ M_2(e_2, m_1') \perp M_1(e_1)$
</td><td>


</td></tr>
</table>

$PIE_{effect13}$

$Definition$

$E\{[Y(0,0,1)-Y(0,0,0)][M_1-M_1(0)][M_2(1,1)-M_2(1,0)-M_2(0,1)+M_2(0,0)]*A\}$

$=E\left[Y\left(0,0,M_2(A,M_1)\right)-Y\left(0,0,M_2(0,M_1)\right)\right]$

$\qquad -E\left[Y\left(0,0,M_2\left(A,M_1(0)\right)\right)-Y\left(0,0,M_2\left(0,M_1(0)\right)\right)\right]$

$=[\phi_{19}(0,0,A,M_1)-\phi_{12}(0,0,0,M_1)]-[\phi_{26}(0,0,A,0)-\phi_3(0,0,0,0)]$

$Identification$

$\phi_3=E\left[Y\left(a',m_1',M_2\left(e_2',M_1(e_1')\right)\right)\right]$

$=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=e_1')$

$\phi_{12}=E\left[Y\left(a',m_1',M_2(e_2',M_1)\right)\right]$

$=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1)$

$\phi_{19}=E\left[Y\left(a',m_1',M_2(A,M_1)\right)\right]$

$=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=a,M_1=m_1)P(M_1=m_1|A=a)P(A=a)$

$\phi_{26}=E\left[Y\left(a',m_1',M_2\left(A,M_1(e_1')\right)\right)\right]$

$=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=a,M_1=m_1)P(M_1=m_1|A=e_1')P(A=a)$

$Required\ Assumptions$

$(A1)\ Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$

- $(A1.1)\ Y(a,m_1,m_2)\perp A$
- $(A1.2)\ Y(a,m_1,m_2)\perp M_1|A$
- $(A1.3)\ Y(a,m_1,m_2)\perp M_2|A,M_1$

$(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$

$(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$

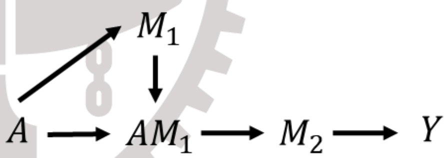
$(A4)\ M_1(e_1)\perp A$

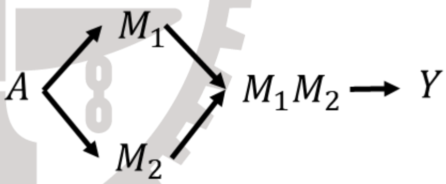
$(A5)\ M_2(e_2,m_1')\perp\{A,M_1\}$

- $(A5.1)\ M_2(e_2,m_1')\perp A$
- $(A5.2)\ M_2(e_2,m_1')\perp M_1|A$

$(A6)\ M_2(e_2,m_1')\perp M_1(e_1)$

DAG



$PIE_{effect14}$	
$Definition$	
$E\{[Y(0,1,1)-Y(0,1,0)-Y(0,0,1)+Y(0,0,0)][M_1-M_1(0)][M_2(1,0)-M_2(0,0)]*A\}$ $=E\big[Y\big(0,M_1,M_2(A,0)\big)-Y\big(0,M_1(0),M_2(A,0)\big)\big]-E\big[Y\big(0,M_1,M_2(0,0)\big)-Y\big(0,M_1(0),M_2(0,0)\big)\big]$ $=[\phi_{20}(0,M_1,A,0)-\phi_{30}(0,0,A,0)]-[\phi_{11}(0,M_1,0,0)-\phi_5(0,0,0,0)]$	
$Identification$	
$\phi_5=E\big[Y\big(a',M_1(e_1'),M_2(e_2',m_1')\big)\big]$ $=\Sigma_{m_1,m_2}E[Y\mid A=a',M_1=m_1,M_2=m_2]P(M_2=m_2\mid A=e_2',M_1=m_1')P(M_1=m_1\mid A=e_1')$ $\phi_{11}=E\big[Y\big(a',M_1,M_2(e_2',m_1')\big)\big]$ $=\Sigma_{m_1,m_2}E[Y\mid A=a',M_1=m_1,M_2=m_2]P(M_2=m_2\mid A=e_2',M_1=m_1')P(M_1=m_1)$ $\phi_{20}=E\big[Y\big(a',M_1,M_2(A,m_1')\big)\big]$ $=\Sigma_{a,m_1,m_2}E[Y\mid A=a',M_1=m_1,M_2=m_2]P(M_2=m_2\mid A=a,M_1=m_1')P(M_1=m_1\mid A=a)P(A=a)$ $\phi_{30}=E\big[Y\big(a',M_1(e_1'),M_2(A,m_1')\big)\big]$ $=\Sigma_{a,m_1,m_2}E[Y\mid A=a',M_1=m_1,M_2=m_2]P(M_2=m_2\mid A=a,M_1=m_1')P(M_1=m_1\mid A=e_1')P(A=a)$	
$Required\ Assumptions$	DAG
$(A1)\ Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$ • $(A1.1)\ Y(a,m_1,m_2)\perp A$ • $(A1.2)\ Y(a,m_1,m_2)\perp M_1\mid A$ • $(A1.3)\ Y(a,m_1,m_2)\perp M_2\mid A,M_1$ $(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$ $(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$ $(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$ $(A4)\ M_1(e_1)\perp A$ $(A5)\ M_2(e_2,m_1')\perp\{A,M_1\}$ • $(A5.1)\ M_2(e_2,m'_1)\perp A$ • $(A5.2)\ M_2(e_2,m_1')\perp M_1\mid A$ $(A6)\ M_2(e_2,m_1')\perp M_1(e_1)$	

$PIE_{effect15}$

$Definition$

$$E\{[Y(0,1,1)-Y(0,1,0)-Y(0,0,1)+Y(0,0,0)]M_1(0)[M_2(1,1)-M_2(0,1)-M_2(1,0)+M_2(0,0)]*A\}$$

$$=E\left[Y\left(0,M_1(0),M_2\big(A,M_1(0)\big)\right)-Y\left(0,M_1(0),M_2\big(0,M_1(0)\big)\right)\right]-E\big[Y\big(0,M_1(0),M_2(A,0)\big)-Y\big(0,M_1(0),M_2(0,0)\big)\big]$$

$$-E\left[Y\left(0,0,M_2\big(A,M_1(0)\big)\right)-Y\left(0,0,M_2\big(0,M_1(0)\big)\right)\right]+E\big[Y\big(0,0,M_2(A,0)\big)-Y\big(0,0,M_2(0,0)\big)\big]$$

$$=[\phi_{33}(0,0,A,0)-\phi_6(0,0,0,0)]-[\phi_{30}(0,0,A,0)-\phi_5(0,0,0,0)]-[\phi_{26}(0,0,A,0)-\phi_3(0,0,0,0)]$$

$$+[\phi_{13}(0,0,A,0)-\phi_2(0,0,0,0)]$$

$Identification$

$$\phi_2=E\big[Y\big(a',m_1',M_2(e_2',m_1')\big)\big]$$

$$=\Sigma_{m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')$$

$$\phi_3=E\left[Y\left(a',m_1',M_2\big(e_2',M_1(e_1')\big)\right)\right]$$

$$=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=e_1')$$

$$\phi_5=E\big[Y\big(a',M_1(e_1'),M_2(e_2',m_1')\big)\big]$$

$$=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(M_1=m_1|A=e_1')$$

$$\phi_6=E\left[Y\left(a',M_1(e_1'),M_2\big(e_2',M_1(e_1')\big)\right)\right]$$

$$=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=e_1')$$

$$\phi_{13}=E\big[Y\big(a',m_1',M_2(A,m_1')\big)\big]$$

$$=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=a,M_1=m_1')P(A=a)$$

$$\phi_{26}=E\left[Y\left(a',m_1',M_2\big(A,M_1(e_1')\big)\right)\right]$$

$$=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=a,M_1=m_1)P(M_1=m_1|A=e_1')P(A=a)$$

$$\phi_{30}=E\left[Y\left(a',m_1',M_2\big(A,M_1(e_1')\big)\right)\right]$$

$$=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=a,M_1=m_1)P(M_1=m_1|A=e_1')P(A=a)$$

$$\phi_{33}=E\left[Y\left(a',M_1(e_1'),M_2\big(A,M_1(e_1')\big)\right)\right]$$

$$=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=a,M_1=m_1)P(M_1=m_1|A=e_1')P(A=a)$$

$Required\ Assumptions$

$(A1)\ Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$

- $(A1.1)\ Y(a,m_1,m_2)\perp A$
- $(A1.2)\ Y(a,m_1,m_2)\perp M_1|A$
- $(A1.3)\ Y(a,m_1,m_2)\perp M_2|A,M_1$

$(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$

$(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$

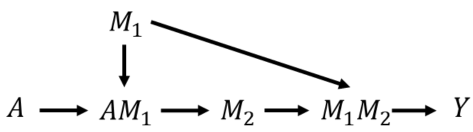
$(A4)\ M_1(e_1)\perp A$

$(A5)\ M_2(e_2,m_1')\perp\{A,M_1\}$

- $(A5.1)\ M_2(e_2,m_1')\perp A$
- $(A5.2)\ M_2(e_2,m_1')\perp M_1|A$

$(A6)\ M_2(e_2,m_1')\perp M_1(e_1)$

DAG



$PIE_{effect16}$

$Definition$

$E\{[Y(0,1,1)-Y(0,1,0)-Y(0,0,1)+Y(0,0,0)]M_1(0)[M_2(1,0)-M_2(0,0)]*A\}$

$=E\big[Y\big(0,M_1(0),M_2(A,0)\big)-Y\big(0,M_1(0),M_2(0,0)\big)\big]-E\big[Y\big(0,0,M_2(A,0)\big)-Y\big(0,0,M_2(0,0)\big)\big]$

$=[\phi_{30}(0,0,A,0)-\phi_5(0,0,0,0)]-[\phi_{13}(0,0,A,0)-\phi_2(0,0,0,0)]$

$Identification$

$\phi_2=E\big[Y\big(a',m_1',M_2(e_2',m_1')\big)\big]$

$=\Sigma_{m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')$

$\phi_5=E\big[Y\big(a',M_1(e_1'),M_2(e_2',m_1')\big)\big]$

$=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(M_1=m_1|A=e_1')$

$\phi_{13}=E\big[Y\big(a',m_1',M_2(A,m_1')\big)\big]$

$=\Sigma_{a,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=a,M_1=m_1')P(A=a)$

$\phi_{30}=E\big[Y\big(a',M_1(e_1'),M_2(A,m_1')\big)\big]$

$=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=a,M_1=m_1')P(M_1=m_1|A=e_1')P(A=a)$

$Required\ Assumptions$

$(A1)\ Y(a,m_1,m_2)\perp\{A\ ,M_1,M_2\}$

- $(A1.1)\ Y(a,m_1,m_2)\perp A$
- $(A1.2)\ Y(a,m_1,m_2)\perp M_1|A$
- $(A1.3)\ Y(a,m_1,m_2)\perp M_2|A,M_1$

$(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$

$(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$

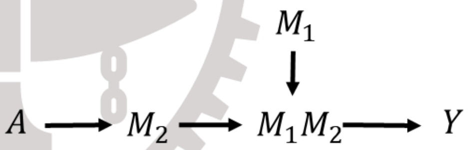
$(A4)\ M_1(e_1)\perp A$

$(A5)\ M_2(e_2,m_1')\perp\{A,M_1\}$

- $(A5.1)\ M_2(e_2,m_1')\perp A$
- $(A5.2)\ M_2(e_2,m_1')\perp M_1|A$

$(A6)\ M_2(e_2,m_1')\perp M_1(e_1)$

DAG



$PIE_{effect17}$

Definition

$E\{[Y(0,1,1)-Y(0,1,0)-Y(0,0,1)+Y(0,0,0)][M_1-M_1(0)][M_2(1,1)-M_2(1,0)-M_2(0,1)+M_2(0,0)]*A\}$

$=\{E[Y(0,M_1,M_2(A,M_1))-Y(0,M_1,M_2(0,M_1))]-E[Y(0,M_1,M_2(A,0))-Y(0,M_1(0),M_2(A,0))]-E[Y(0,0,M_2(A,M_1))-Y(0,0,M_2(0,M_1))]\}$

$-\{E\left[Y\left(0,M_1(0),M_2(A,M_1(0))\right)-Y\left(0,M_1(0),M_2(0,M_1(0))\right)\right]-E[Y(0,M_1,M_2(0,0))-Y(0,M_1(0),M_2(0,0))]$

$-E\left[Y\left(0,0,M_2(A,M_1(0))\right)-Y\left(0,0,M_2(0,M_1(0))\right)\right]\}$

$=\{[\phi_{21}(0,M_1,A,M_1)-\phi_{17}(0,M_1,0,M_1)]-[\phi_{20}(0,M_1,A,0)-\phi_{30}(0,0,A,0)]-[\phi_{19}(0,0,A,M_1)-\phi_{12}(0,0,0,M_1)]\}-\{[\phi_{33}(0,0,A,0)-\phi_6(0,0,0,0)]$

$-[\phi_{11}(0,M_1,0,0)-\phi_5(0,0,0,0)]-[\phi_{26}(0,0,A,0)-\phi_3(0,0,0,0)]\}$

Identification

$\phi_3=E\left[Y\left(a',m_1',M_2(e_2',M_1(e_1'))\right)\right]=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=e_1')$

$\phi_5=E[Y(a',M_1(e_1'),M_2(e_2',m_1'))]=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(M_1=m_1|A=e_1')$

$\phi_6=E\left[Y\left(a',M_1(e_1'),M_2(e_2',M_1(e_1'))\right)\right]=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=e_1')$

$\phi_{11}=E[Y(a',M_1,M_2(e_2',m_1'))]=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(M_1=m_1)$

$\phi_{12}=E[Y(a',m_1',M_2(e_2',M_1))]=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1)$

$\phi_{17}=E[Y(a',M_1,M_2(e_2',M_1))]=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1)$

$\phi_{19}=E[Y(a',m_1',M_2(A,M_1))]=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=a,M_1=m_1)P(M_1=m_1|A=a)P(A=a)$

$\phi_{20}=E[Y(a',M_1,M_2(A,m_1'))]=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=a,M_1=m_1')P(M_1=m_1|A=a)P(A=a)$

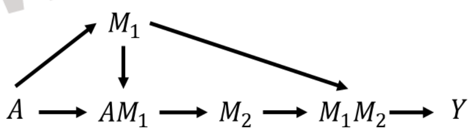
$\phi_{21}=E[Y(a',M_1,M_2(A,M_1))]=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=a,M_1=m_1)P(M_1=m_1|A=a)P(A=a)$

$\phi_{26}=E[Y(a',M_1(e_1'),M_2(A,m_1'))]=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=a,M_1=m_1')P(M_1=m_1|A=e_1')P(A=a)$

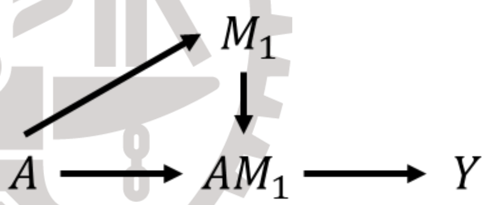
$\phi_{30}=E[Y(a',M_1(e_1'),M_2(A,m_1'))]=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=a,M_1=m_1')P(M_1=m_1|A=e_1')P(A=a)$

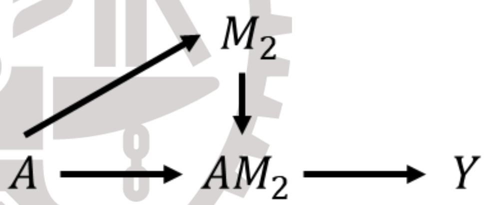
$\phi_{33}=E\left[Y\left(a',M_1(e_1'),M_2(A,M_1(e_1'))\right)\right]=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=a,M_1=m_1)P(M_1=m_1|A=e_1')P(A$

$=a)$

Required Assumptions	*DAG*
$(A1)\ Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$ • $(A1.1)\ Y(a,m_1,m_2)\perp A$ • $(A1.2)\ Y(a,m_1,m_2)\perp M_1\vert A$ • $(A1.3)\ Y(a,m_1,m_2)\perp M_2\vert A,M_1$ $(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$ $(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$ $(A4)\ M_1(e_1)\perp A$ $(A5)\ M_2(e_2,m'_1)\perp\{A,M_1\}$ • $(A5.1)\ M_2(e_2,m_1')\perp A$ • $(A5.2)\ M_2(e_2,m_1')\perp M_1\vert A$ $(A6)\ M_2(e_2,m_1')\perp M_1(e_1)$	

<table>
<tr><th colspan="2">$PIE_{effect18}$</th></tr>
<tr><th colspan="2">$Definition$</th></tr>
<tr><td colspan="2">$E\{[Y(1,1,0) - Y(1,0,0) - Y(0,1,0) + Y(0,0,0)][M_1 - M_1(0)] * A\}$
$= E[Y(A, M_1, 0) - Y(0, M_1, 0)] - E[Y(A, M_1(0), 0) - Y(0, M_1(0), 0)]$
$= [\phi_8(A, M_1, 0) - \phi_9(0, M_1, 0)] - [\phi_{28}(A, 0,0) - \phi_4(0,0,0)]$</td></tr>
<tr><th colspan="2">$Identification$</th></tr>
<tr><td colspan="2">$\phi_4 = E[Y(a', M_1(e_1'), m_2')]$
$= \Sigma_{m_1} E[Y \mid A = a', M_1 = m_1, M_2 = m_2'] P(M_1 = m_1 \mid A = e_1')$
$\phi_8 = E[Y(A, M_1, m_2')]$
$= \Sigma_{a,m_1} E[Y \mid A = a, M_1 = m_1, M_2 = m_2'] P(M_1 = m_1 \mid A = a) P(A = a)$
$\phi_9 = E[Y(a', M_1, m_2')]$
$= \Sigma_{m_1} E[Y \mid A = a', M_1 = m_1, M_2 = m_2'] P(M_1 = m_1)$
$\phi_{28} = E[Y(A, M_1(e_1'), m_2')]$
$= \Sigma_{a,m_1} E[Y \mid A = a, M_1 = m_1, M_2 = m_2'] P(M_1 = m_1 \mid A = e_1') P(A = a)$</td></tr>
<tr><th>$Required\ Assumptions$</th><th>DAG</th></tr>
<tr><td>$(A1)\ Y(a, m_1, m_2) \perp \{A, M_1, M_2\}$
• $(A1.1)\ Y(a, m_1, m_2) \perp A$
• $(A1.2)\ Y(a, m_1, m_2) \perp M_1 \mid A$
• $(A1.3)\ Y(a, m_1, m_2) \perp M_2 \mid A, M_1$
$(A2)\ Y(a, m_1, m_2) \perp M_1(e_1)$
$(A3)\ Y(a, m_1, m_2) \perp M_2(e_2, m_1')$
$(A4)\ M_1(e_1) \perp A$
$(A5)\ M_2(e_2, m'_1) \perp \{A, M_1\}$
• $(A5.1)\ M_2(e_2, m_1') \perp A$
• $(A5.2)\ M_2(e_2, m_1') \perp M_1 \mid A$

$(A6)\ M_2(e_2, m_1') \perp M_1(e_1)$</td><td></td></tr>
</table>

<table>
<tr><th colspan="2">$PIE_{effect19}$</th></tr>
<tr><td colspan="2" align="center">$Definition$</td></tr>
<tr><td colspan="2">$E\{[Y(1,0,1) - Y(1,0,0) - Y(0,0,1) + Y(0,0,0)][M_2(1,0) - M_2(0,0)] * A\}$
$= E\left[Y\left(A, 0, M_2(A, 0)\right) - Y\left(A, 0, M_2(0,0)\right)\right]$
$- E\left[Y\left(0,0, M_2(A, 0)\right) - Y\left(0,0, M_2(0,0)\right)\right]$
$= [\phi_{18}(A, 0, A, 0) - \phi_{10}(A, 0,0,0)] - [\phi_{13}(0,0, A, 0) - \phi_2(0,0,0,0)]$</td></tr>
<tr><td colspan="2" align="center">$Identification$</td></tr>
<tr><td colspan="2">$\phi_2 = E\left[Y\left(a', m_1', M_2(e_2', m_1')\right)\right]$
$= \Sigma_{m_2} E[Y \mid A = a', M_1 = m_1', M_2 = m_2] P(M_2 = m_2 \mid A = e_2', M_1 = m_1')$
$\phi_{10} = E\left[Y\left(A, m_1', M_2(e_2', m_1')\right)\right]$
$= \Sigma_{a,m_2} E[Y \mid A = a, M_1 = m_1', M_2 = m_2] P(M_2 = m_2 \mid A = e_2', M_1 = m_1') P(A = a)$
$\phi_{13} = E\left[Y\left(a', m_1', M_2(A, m_1')\right)\right]$
$= \Sigma_{a,m_2} E[Y \mid A = a', M_1 = m_1', M_2 = m_2] P(M_2 = m_2 \mid A = a, M_1 = m_1') P(A = a)$
$\phi_{18} = E\left[Y\left(A, m_1', M_2(A, m_1')\right)\right]$
$= \Sigma_{a,m_2} E[Y \mid A = a, M_1 = m_1', M_2 = m_2] P(M_2 = m_2 \mid A = a, M_1 = m_1') P(A = a)$</td></tr>
<tr><td align="center">$Required\ Assumptions$</td><td align="center">DAG</td></tr>
<tr><td>$(A1)\ Y(a, m_1, m_2) \perp \{A, M_1, M_2\}$
• $(A1.1)\ Y(a, m_1, m_2) \perp A$
• $(A1.2)\ Y(a, m_1, m_2) \perp M_1 \mid A$
• $(A1.3)\ Y(a, m_1, m_2) \perp M_2 \mid A, M_1$
$(A2)\ Y(a, m_1, m_2) \perp M_1(e_1)$
$(A3)\ Y(a, m_1, m_2) \perp M_2(e_2, m_1')$
$(A5)\ M_2(e_2, m_1') \perp \{A, M_1\}$
• $(A5.1)\ M_2(e_2, m_1') \perp A$
• $(A5.2)\ M_2(e_2, m_1') \perp M_1 \mid A$</td><td>
</td></tr>
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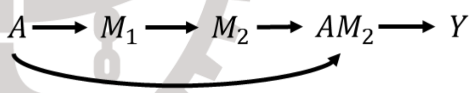
<table>
<tr><th colspan="2">$PIE_{effect20}$</th></tr>
<tr><th colspan="2">$Definition$</th></tr>
<tr><td colspan="2">$E\{[Y(1,0,1)-Y(1,0,0)-Y(0,0,1)+Y(0,0,0)][M_2(0,1)-M_2(0,0)][M_1-M_1(0)]*A\}$
$=E\left[Y\big(A,0,M_2(0,M_1)\big)-Y\Big(A,0,M_2\big(0,M_1(0)\big)\Big)\right]$
$-E\left[Y\big(0{,}0,M_2(0,M_1)\big)-Y\Big(0{,}0,M_2\big(0,M_1(0)\big)\Big)\right]$
$=[\phi_{12}(A,0{,}0,M_1)-\phi_{15}(A,0{,}0{,}0)]-[\phi_{11}(0{,}0{,}0,M_1)-\phi_3(0{,}0{,}0{,}0)]$</td></tr>
<tr><th colspan="2">$Identification$</th></tr>
<tr><td colspan="2">$\phi_3=E\left[Y\Big(a',m_1',M_2\big(e_2',M_1(e_1')\big)\Big)\right]$
$=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=e_1')$
$\phi_{11}=E\big[Y\big(a',m_1',M_2(e_2',M_1)\big)\big]$
$=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1)$
$\phi_{12}=E\big[Y\big(A,m_1',M_2(e_2',M_1)\big)\big]$
$=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=a)P(A=a)$
$\phi_{15}=E\left[Y\Big(A,m_1',M_2\big(e_2',M_1(e_1')\big)\Big)\right]$
$=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=e_1')P(A=a)$</td></tr>
<tr><th>$Required\ Assumptions$</th><th>DAG</th></tr>
<tr><td>$(A1)\ Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$
● $(A1.1)\ Y(a,m_1,m_2)\perp A$
● $(A1.2)\ Y(a,m_1,m_2)\perp M_1|A$
● $(A1.3)\ Y(a,m_1,m_2)\perp M_2|A,M_1$
$(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$
$(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$
$(A4)\ M_1(e_1)\perp A$
$(A5)\ M_2(e_2,m_1')\perp\{A,M_1\}$
● $(A5.1)\ M_2(e_2,m_1')\perp A$
● $(A5.2)\ M_2(e_2,m_1')\perp M_1|A$
$(A6)\ M_2(e_2,m_1)\perp M_1(e_1)$</td><td></td></tr>
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$PIE_{effect21}$

$Definition$

$$E\{[Y(1,1,1)-Y(1,1,0)-Y(1,0,1)+Y(1,0,0)-Y(0,1,1)+Y(0,1,0)+Y(0,0,1)-Y(0,0,0)][M_1-M_1(0)]M_2(0,0)*A\}$$

$$=E\left[Y\left(A,M_1,M_2(0,0)\right)-Y\left(A,M_1,M_2(0,0)\right)\right]-E\left[Y\left(0,M_1,M_2(0,0)\right)-Y\left(0,M_1(0),M_2(0,0)\right)\right]-E[Y(A,M_1,0)-Y(0,M_1,0)]+E[Y(A,M_1(0),0)-Y(0,M_1(0),0)]$$

$$=[\phi_{22}(A,M_1,0,0)-\phi_{25}(A,0,0,0)]-[\phi_{21}(0,M_1,0,0)-\phi_5(0,0,0,0)]-[\phi_{18}(A,M_1,0)-\phi_{19}(0,M_1,0)]+[\phi_{20}(A,0,0)-\phi_4(0,0,0)]$$

$Identification$

$\phi_4=E[Y(a',M_1(e_1'),m_2')]=\Sigma_{m_1}E[Y|A=a',M_1=m_1,M_2=m_2']P(M_1=m_1|A=e_1')$

$\phi_5=E\left[Y\left(a',M_1(e_1'),M_2(e_2',m_1')\right)\right]$

$=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(M_1=m_1|A=e_1')$

$\phi_{18}=E[Y(A,M_1,m_2')]$

$=\Sigma_{a,m_1}E[Y|A=a,M_1=m_1,M_2=m_2']P(M_1=m_1|A=a)P(A=a)$

$\phi_{19}=E[Y(a',M_1,m_2')]$

$=\Sigma_{m_1}E[Y|A=a',M_1=m_1,M_2=m_2']P(M_1=m_1)$

$\phi_{20}=E[Y(A,M_1(e_1'),m_2')]$

$=\Sigma_{a,m_1}E[Y|A=a,M_1=m_1,M_2=m_2']P(M_1=m_1|A=e_1')P(A=a)$

$\phi_{21}=E\left[Y\left(a',M_1,M_2(e_2',m_1')\right)\right]$

$=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(M_1=m_1)$

$\phi_{22}=E\left[Y\left(A,M_1,M_2(e_2',m_1')\right)\right]$

$=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(M_1=m_1|A=a)P(A=a)$

$\phi_{25}=E\left[Y\left(A,M_1(e_1'),M_2(e_2',m_1')\right)\right]$

$=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(M_1=m_1|A=e_1')P(A=a)$

$Required\ Assumptions$

$(A1)$ $Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$

- $(A1.1)$ $Y(a,m_1,m_2)\perp A$
- $(A1.2)$ $Y(a,m_1,m_2)\perp M_1|A$
- $(A1.3)$ $Y(a,m_1,m_2)\perp M_2|A,M_1$

$(A2)$ $Y(a,m_1,m_2)\perp M_1(e_1)$

$(A3)$ $Y(a,m_1,m_2)\perp M_2(e_2,m_1')$

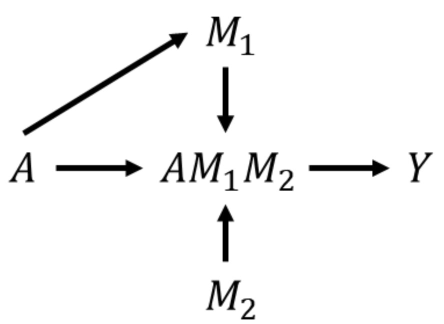
$(A4)$ $M_1(e_1)\perp A$

$(A5)$ $M_2(e_2,m_1')\perp\{A,M_1\}$

- $(A5.1)$ $M_2(e_2,m_1')\perp A$
- $(A5.2)$ $M_2(e_2,m_1')\perp M_1|A$

$(A6)$ $M_2(e_2,m_1')\perp M_1(e_1)$

DAG



$PIE_{effect22}$

Definition

$$E\{[Y(1,1,1)-Y(1,1,0)-Y(1,0,1)+Y(1,0,0)-Y(0,1,1)+Y(0,1,0)+Y(0,0,1)-Y(0,0,0)][M_1-M_1(0)][M_2(0,1)-M_2(0,0)]*A\}$$

$$=\left\{E\left[Y(A,M_1,M_2(0,M_1))-Y\left(A,M_1(0),M_2(0,M_1(0))\right)\right]-E[Y(A,M_1,M_2(0,0))-Y(A,M_1(0),M_2(0,0))]\right.$$

$$\left.-E\left[Y(A,0,M_2(0,M_1))-Y\left(A,0,M_2(0,M_1(0))\right)\right]\right\}$$

$$-\left\{E\left[Y(0,M_1,M_2(0,M_1))-Y\left(0,M_1(0),M_2(0,M_1(0))\right)\right]-E[Y(0,M_1,M_2(0,0))-Y(0,M_1(0),M_2(0,0))]\right.$$

$$\left.-E\left[Y(0,0,M_2(0,M_1))-Y\left(0,0,M_2(0,M_1(0))\right)\right]\right\}$$

$$=\{[\phi_{28}(A,M_1,0,M_1)-\phi_{31}(A,0,0,0)]-[\phi_{22}(A,M_1,0,0)-\phi_{25}(A,0,0,0)]-[\phi_{12}(A,0,0,M_1)-\phi_{15}(A,0,0,0)]\}$$

$$-\{[\phi_{29}(0,M_1,0,M_1)-\phi_6(0,0,0,0)]-[\phi_{21}(0,M_1,0,0)-\phi_5(0,0,0,0)]-[\phi_{11}(0,0,0,M_1)-\phi_3(0,0,0,0)]\}$$

Identification

$$\phi_3=E\left[Y\left(a',m_1',M_2(e_2',M_1(e_1'))\right)\right]=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=e_1')$$

$$\phi_5=E[Y(a',M_1(e_1'),M_2(e_2',m_1'))]=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(M_1=m_1|A=e_1')$$

$$\phi_6=E\left[Y\left(a',M_1(e_1'),M_2(e_2',M_1(e_1'))\right)\right]=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=e_1')$$

$$\phi_{11}=E[Y(a',m_1',M_2(e_2',M_1))]=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=a)P(A=a)$$

$$\phi_{12}=E[Y(A,m_1',M_2(e_2',M_1))]=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=a)P(A=a)$$

$$\phi_{15}=E\left[Y\left(A,m_1',M_2(e_2',M_1(e_1'))\right)\right]=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=e_1')P(A=a)$$

$$\phi_{21}=E[Y(a',M_1,M_2(e_2',m_1'))]=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(M_1=m_1|A=a)P(A=a)$$

$$\phi_{22}=E[Y(A,M_1,M_2(e_2',m_1'))]=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(M_1=m_1|A=a)P(A=a)$$

$$\phi_{25}=E[Y(A,M_1(e_1'),M_2(e_2',m_1'))]=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(M_1=m_1|A=e_1')P(A=a)$$

$$\phi_{28}=E[Y(A,M_1,M_2(e_2',M_1))]=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=a)P(A=a)$$

$$\phi_{29}=E[Y(a',M_1,M_2(e_2',M_1))]=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=a)P(A=a)$$

$$\phi_{31}=E\left[Y\left(A,M_1(e_1'),M_2(e_2',M_1(e_1'))\right)\right]=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=e_1')P(A=a)$$

Required Assumptions

(A1) $Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$

- (A1.1) $Y(a,m_1,m_2)\perp A$
- (A1.2) $Y(a,m_1,m_2)\perp M_1|A$
- (A1.3) $Y(a,m_1,m_2)\perp M_2|A,M_1$

(A2) $Y(a,m_1,m_2)\perp M_1(e_1)$

(A3) $Y(a,m_1,m_2)\perp M_2(e_2,m_1')$

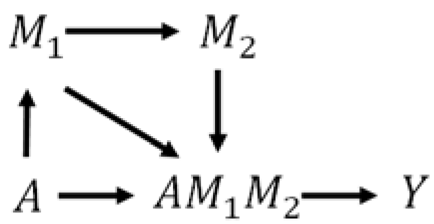
(A4) $M_1(e_1)\perp A$

(A5) $M_2(e_2,m_1')\perp\{A,M_1\}$

- (A5.1) $M_2(e_2,m_1')\perp A$
- (A5.2) $M_2(e_2,m_1')\perp M_1|A$

(A6) $M_2(e_2,m_1')\perp M_1(e_1)$

DAG



$PIE_{effect23}$
$Definition$

$$E\{[Y(1,0,1)-Y(1,0,0)-Y(0,0,1)+Y(0,0,0)][M_1-M_1(0)][M_2(1,1)-M_2(1,0)-M_2(0,1)+M_2(0,0)]*A\}$$

$$=E\left[Y\big(A,0,M_2(A,M_1)\big)-Y\Big(A,0,M_2\big(A,M_1(0)\big)\Big)\right]-E\left[Y\big(A,0,M_2(0,M_1)\big)-Y\Big(A,0,M_2\big(0,M_1(0)\big)\Big)\right]$$

$$-E\left[Y\big(0,0,M_2(A,M_1)\big)-Y\Big(0,0,M_2\big(A,M_1(0)\big)\Big)\right]+E\left[Y\big(0,0,M_2(0,M_1)\big)-Y\Big(0,0,M_2\big(0,M_1(0)\big)\Big)\right]$$

$$=[\phi_{14}(A,0,A,M_1)-\phi_{17}(A,0,A,0)]-[\phi_{12}(A,0,0,M_1)-\phi_{15}(A,0,0,0)]-[\phi_{13}(0,0,A,M_1)-\phi_{16}(0,0,A,0)]$$

$$+[\phi_{11}(0,0,0,M_1)-\phi_3(0,0,0,0)]$$

$Identification$

$$\phi_3=E\left[Y\Big(a',m_1',M_2\big(e_2',M_1(e_1')\big)\Big)\right]=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=e_1')$$

$$\phi_{11}=E\big[Y\big(a',m_1',M_2(e_2',M_1)\big)\big]$$

$$=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1)$$

$$\phi_{12}=E\big[Y\big(A,m_1',M_2(e_2',M_1)\big)\big]$$

$$=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=a)P(A=a)$$

$$\phi_{13}=E\big[Y\big(a',m_1',M_2(A,M_1)\big)\big]$$

$$=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=a,M_1=m_1)P(M_1=m_1|A=a)P(A=a)$$

$$\phi_{14}=E\big[Y\big(A,m_1',M_2(A,M_1)\big)\big]$$

$$=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1',M_2=m_2]P(M_2=m_2|A=a,M_1=m_1)P(M_1=m_1|A=a)P(A=a)$$

$$\phi_{15}=E\left[Y\Big(A,m_1',M_2\big(e_2',M_1(e_1')\big)\Big)\right]$$

$$=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=e_1')P(A=a)$$

$$\phi_{16}=E\left[Y\Big(a',m_1',M_2\big(A,M_1(e_1')\big)\Big)\right]$$

$$=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=a,M_1=m_1)P(M_1=m_1|A=e_1')P(A=a)$$

$$\phi_{17}=E\left[Y\Big(A,m_1',M_2\big(A,M_1(e_1')\big)\Big)\right]$$

$$=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1',M_2=m_2]P(M_2=m_2|A=a,M_1=m_1)P(M_1=m_1|A=e_1')P(A=a)$$

$Required\ Assumptions$	DAG

$(A1)$ $Y(a,m_1,m_2)\perp\{A\,,M_1,M_2\}$

- $(A1.1)$ $Y(a,m_1,m_2)\perp A$
- $(A1.2)$ $Y(a,m_1,m_2)\perp M_1|A$
- $(A1.3)$ $Y(a,m_1,m_2)\perp M_2|A,M_1$

$(A2)$ $Y(a,m_1,m_2)\perp M_1(e_1)$

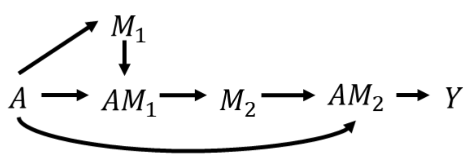
$(A3)$ $Y(a,m_1,m_2)\perp M_2(e_2,m_1')$

$(A4)$ $M_1(e_1)\perp A$

$(A5)$ $M_2(e_2,m_1')\perp\{A,M_1\}$

- $(A5.1)$ $M_2(e_2,m_1')\perp A$
- $(A5.2)$ $M_2(e_2,m_1')\perp M_1|A$

$(A6)$ $M_2(e_2,m_1')\perp M_1(e_1)$



$PIE_{effect24}$

$Definition$

$$E\{[Y(1,0,1)-Y(1,0,0)-Y(0,0,1)+Y(0,0,0)]M_1(0)[M_2(1,1)-M_2(1,0)-M_2(0,1)+M_2(0,0)]*A\}$$
$$=E\left[Y\left(A,0,M_2(A,M_1(0))\right)-Y\left(A,0,M_2(0,M_1(0))\right)\right]-E\left[Y\left(A,0,M_2(A,0)\right)-Y\left(A,0,M_2(0,0)\right)\right]$$
$$-E\left[Y\left(0,0,M_2(A,M_1(0))\right)-Y\left(0,0,M_2(0,M_1(0))\right)\right]$$
$$+E\left[Y\left(0,0,M_2(A,0)\right)-Y\left(0,0,M_2(0,0)\right)\right]$$
$$=[\phi_{17}(A,0,A,0)-\phi_{15}(A,0,0,0)]-[\phi_{10}(A,0,A,0)-\phi_8(A,0,0,0)]-[\phi_{16}(0,0,A,0)-\phi_3(0,0,0,0)]$$
$$+[\phi_9(0,0,A,0)-\phi_2(0,0,0,0)]$$

$Identification$

$$\phi_2=E\left[Y\left(a',m_1',M_2(e_2',m_1')\right)\right]=\Sigma_{m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')$$

$$\phi_3=E\left[Y\left(a',m_1',M_2(e_2',M_1(e_1'))\right)\right]$$
$$=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=e_1')$$

$$\phi_8=E\left[Y\left(A,m_1',M_2(e_2',m_1')\right)\right]$$
$$=\Sigma_{a,m_2}E[Y|A=a,M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(A=a)$$

$$\phi_9=E\left[Y\left(a',m_1',M_2(A,m_1')\right)\right]$$
$$=\Sigma_{a,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=a,M_1=m_1')P(A=a)$$

$$\phi_{10}=E\left[Y\left(A,m_1',M_2(A,m_1')\right)\right]$$
$$=\Sigma_{a,m_2}E[Y|A=a,M_1=m_1',M_2=m_2]P(M_2=m_2|A=a,M_1=m_1')P(A=a)$$

$$\phi_{15}=E\left[Y\left(A,m_1',M_2(e_2',M_1(e_1'))\right)\right]$$
$$=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=e_1')P(A=a)$$

$$\phi_{16}=E\left[Y\left(a',m_1',M_2(A,M_1(e_1'))\right)\right]$$
$$=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=a,M_1=m_1)P(M_1=m_1|A=e_1')P(A=a)$$

$$\phi_{17}=E\left[Y\left(A,m_1',M_2(A,M_1(e_1'))\right)\right]$$
$$=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1',M_2=m_2]P(M_2=m_2|A=a,M_1=m_1)P(M_1=m_1|A=e_1')P(A=a)$$

$Required\ Assumptions$

$(A1)\ Y(a,m_1,m_2)\perp\{A\ ,M_1,M_2\}$
- $(A1.1)\ Y(a,m_1,m_2)\perp A$
- $(A1.2)\ Y(a,m_1,m_2)\perp M_1|A$
- $(A1.3)\ Y(a,m_1,m_2)\perp M_2|A,M_1$

$(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$

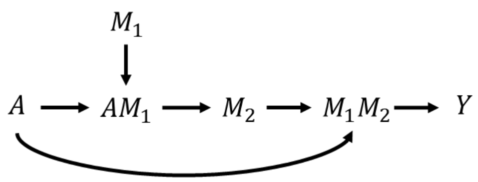
$(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$

$(A4)\ M_1(e_1)\perp A$

$(A5)\ M_2(e_2,m_1')\perp\{A,M_1\}$
- $(A5.1)\ M_2(e_2,m_1')\perp A$
- $(A5.2)\ M_2(e_2,m_1')\perp M_1|A$

$(A6)\ M_2(e_2,m_1')\perp M_1(e_1)$

DAG



$PIE_{effect25}$

$Definition$

$$E\{[Y(1,1,1)-Y(1,1,0)-Y(1,0,1)+Y(1,0,0)-Y(0,1,1)+Y(0,1,0)+Y(0,0,1)-Y(0,0,0)][M_1-M_1(0)][M_2(1,0) - M_2(0,0)]*A\}$$

$$= E\big[Y\big(A,M_1,M_2(A,0)\big)-Y\big(A,M_1(0),M_2(A,0)\big)\big]-E\big[Y\big(A,M_1,M_2(0,0)\big)-Y\big(A,M_1(0),M_2(0,0)\big)\big] - E\big[Y\big(0,M_1,M_2(A,0)\big)-Y\big(0,M_1(0),M_2(A,0)\big)\big]+E\big[Y\big(0,M_1,M_2(0,0)\big)-Y\big(0,M_1(0),M_2(0,0)\big)\big]$$

$$= [\phi_{24}(A,M_1,A,0)-\phi_{27}(A,0,A,0)]-[\phi_{22}(A,M_1,0,0)-\phi_{25}(A,0,0,0)]-[\phi_{23}(0,M_1,A,0)-\phi_{26}(0,0,A,0)] + [\phi_{21}(0,M_1,0,0)-\phi_5(0,0,0,0)]$$

$Identification$

$$\phi_5 = E\big[Y\big(a',M_1(e_1'),M_2(e_2',m_1')\big)\big] = \Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(M_1=m_1|A=e_1')$$

$$\phi_{21} = E\big[Y\big(a',M_1,M_2(e_2',m_1')\big)\big] = \Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(M_1=m_1|A=a)P(A=a)$$

$$\phi_{22} = E\big[Y\big(A,M_1,M_2(e_2',m_1')\big)\big] = \Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(M_1=m_1|A=a)P(A=a)$$

$$\phi_{23} = E\big[Y\big(a',M_1,M_2(A,m_1')\big)\big] = \Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=a,M_1=m_1')P(M_1=m_1|A=a)P(A=a)$$

$$\phi_{24} = E\big[Y\big(A,M_1,M_2(A,m_1')\big)\big] = \Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=a,M_1=m_1')P(M_1=m_1|A=a)P(A=a)$$

$$\phi_{25} = E\big[Y\big(A,M_1(e_1'),M_2(e_2',m_1')\big)\big] = \Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(M_1=m_1|A=e_1')P(A=a)$$

$$\phi_{26} = E\big[Y\big(a',M_1(e_1'),M_2(A,m_1')\big)\big] = \Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=a,M_1=m_1')P(M_1=m_1|A=e_1')P(A=a)$$

$$\phi_{27} = E\big[Y\big(A,M_1(e_1'),M_2(A,m_1')\big)\big] = \Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=a,M_1=m_1')P(M_1=m_1|A=e_1')P(A=a)$$

$Required\ Assumptions$

$(A1)$ $Y(a,m_1,m_2) \perp \{A\ ,M_1,M_2\}$

- $(A1.1)$ $Y(a,m_1,m_2) \perp A$
- $(A1.2)$ $Y(a,m_1,m_2) \perp M_1|A$
- $(A1.3)$ $Y(a,m_1,m_2) \perp M_2|A,M_1$

$(A2)$ $Y(a,m_1,m_2) \perp M_1(e_1)$

$(A3)$ $Y(a,m_1,m_2) \perp M_2(e_2,m_1')$

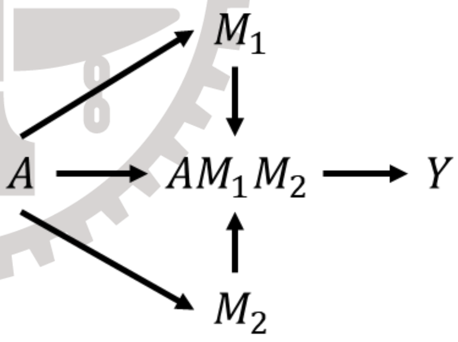
$(A4)$ $M_1(e_1) \perp A$

$(A5)$ $M_2(e_2,m_1') \perp \{A,M_1\}$

- $(A5.1)$ $M_2(e_2,m_1') \perp A$
- $(A5.2)$ $M_2(e_2,m_1') \perp M_1|A$

$(A6)$ $M_2(e_2,m_1') \perp M_1(e_1)$

DAG



$PIE_{effect26}$

$Definition$

$$E\{[Y(1,1,1)-Y(1,1,0)-Y(1,0,1)+Y(1,0,0)-Y(0,1,1)+Y(0,1,0)+Y(0,0,1)-Y(0,0,0)][M_1-M_1(0)][M_2(1,1)-M_2(1,0)-M_2(1,0)+M_2(0,0)]*A\}$$

$$=\Big\{E[Y-Y(A,M_1,M_2(0,M_1))]-E[Y(A,M_1,M_2(A,0))-Y(A,M_1,M_2(0,0))]-E\Big[Y\Big(A,M_1(0),M_2\big(A,M_1(0)\big)\Big)-Y\Big(A,M_1(0),M_2\big(0,M_1(0)\big)\Big)\Big]+E[Y(A,M_1(0),M_2(A,0))-Y(A,M_1(0),M_2(0,0))]-E\Big[Y\big(A,0,M_2(A,M_1)\big)-Y\Big(A,0,M_2\big(A,M_1(0)\big)\Big)\Big]$$

$$+E\Big[Y\big(A,0,M_2(0,M_1)\big)-Y\Big(A,0,M_2\big(0,M_1(0)\big)\Big)\Big]\Big\}$$

$$-\Big\{E[Y(0,M_1,M_2(A,M_1))-Y(0,M_1,M_2(0,M_1))]-E[Y(0,M_1,M_2(A,0))-Y(0,M_1,M_2(0,0))]-E\Big[Y\Big(0,M_1(0),M_2\big(A,M_1(0)\big)\Big)-Y\Big(0,M_1(0),M_2\big(0,M_1(0)\big)\Big)\Big]+E[Y(0,M_1(0),M_2(A,0))-Y(0,M_1(0),M_2(0,0))]$$

$$-E\Big[Y\big(0,0,M_2(A,M_1)\big)-Y\Big(0,0,M_2\big(A,M_1(0)\big)\Big)\Big]+E\Big[Y\big(0,0,M_2(0,M_1)\big)-Y\Big(0,0,M_2\big(0,M_1(0)\big)\Big)\Big]\Big\}$$

$$=\{[E[Y]-\phi_{28}(A,M_1,0,M_1)]-[\phi_{24}(A,M_1,A,0)-\phi_{22}(A,M_1,0,0)]-[\phi_{33}(A,0,A,0)-\phi_{31}(A,0,0,0)]+[\phi_{27}(A,0,A,0)-\phi_{25}(A,0,0,0)]-[\phi_{14}(A,0,A,M_1)-\phi_{17}(A,0,A,0)]+[\phi_{12}(A,0,0,M_1)-\phi_{15}(A,0,0,0)]\}$$

$$-\{[\phi_{30}(0,M_1,A,M_1)-\phi_{29}(0,M_1,0,M_1)]-[\phi_{23}(0,M_1,A,0)-\phi_{21}(0,M_1,0,0)]-[\phi_{32}(0,0,A,0)-\phi_6(0,0,0,0)]+[\phi_{26}(0,0,A,0)-\phi_5(0,0,0,0)]-[\phi_{13}(0,0,A,M_1)-\phi_{16}(0,0,A,0)]+[\phi_{11}(0,0,0,M_1)-\phi_3(0,0,0,0)]\}$$

$Identification$

$\phi_3=E\Big[Y\Big(a',m_1',M_2\big(e_2',M_1(e_1')\big)\Big)\Big]=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=e_1')$

$\phi_5=E\big[Y\big(a',M_1(e_1'),M_2(e_2',m_1')\big)\big]=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(M_1=m_1|A=e_1')$

$\phi_6=E\Big[Y\Big(a',M_1(e_1'),M_2\big(e_2',M_1(e_1')\big)\Big)\Big]=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=e_1')$

$\phi_{11}=E\big[Y\big(a',m_1',M_2(e_2',M_1)\big)\big]=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=a)P(A=a)$

$\phi_{13}=E\big[Y\big(a',m_1',M_2(A,M_1)\big)\big]=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=a,M_1=m_1)P(M_1=m_1|A=a)P(A=a)$

$\phi_{14}=E\big[Y\big(A,m_1',M_2(A,M_1)\big)\big]=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1',M_2=m_2]P(M_2=m_2|A=a,M_1=m_1)P(M_1=m_1|A=a)P(A=a)$

$\phi_{15}=E\Big[Y\Big(A,m_1',M_2\big(e_2',M_1(e_1')\big)\Big)\Big]=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=e_1')P(A=a)$

$\phi_{17}=E\Big[Y\Big(A,m_1',M_2\big(A,M_1(e_1')\big)\Big)\Big]=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1',M_2=m_2]P(M_2=m_2|A=a,M_1=m_1)P(M_1=m_1|A=e_1')P(A=a)$

$\phi_{21}=E\big[Y\big(a',M_1,M_2(e_2',m_1')\big)\big]=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(M_1=m_1|A=a)P(A=a)$

$\phi_{23}=E\big[Y\big(a',M_1,M_2(A,m_1')\big)\big]=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=a,M_1=m_1')P(M_1=m_1|A=a)P(A=a)$

$\phi_{26}=E\big[Y\big(a',M_1(e_1'),M_2(A,m_1')\big)\big]=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=a,M_1=m_1')P(M_1=m_1|A=e_1')P(A=a)$

$\phi_{29}=E\big[Y\big(a',M_1,M_2(e_2',M_1)\big)\big]=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=a)P(A=a)$

$\phi_{30}=E\big[Y\big(a',M_1,M_2(A,M_1)\big)\big]=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=a,M_1=m_1)P(M_1=m_1|A=a)P(A=a)$

$\phi_{32}=E\Big[Y\Big(a',M_1(e_1'),M_2\big(A,M_1(e_1')\big)\Big)\Big]=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=a,M_1=m_1)P(M_1=m_1|A=e_1')P(A=a)$

$Required\ Assumptions$

$(A1)\ Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$

- $(A1.1)\ Y(a,m_1,m_2)\perp A$
- $(A1.2)\ Y(a,m_1,m_2)\perp M_1|A$
- $(A1.3)\ Y(a,m_1,m_2)\perp M_2|A,M_1$

$(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$

$(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$

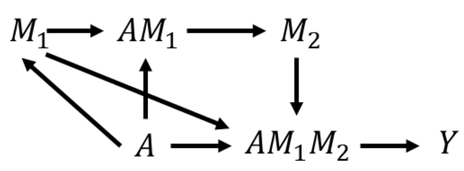
$(A4)\ M_1(e_1)\perp A$

$(A5)\ M_2(e_2,m_1')\perp\{A,M_1\}$

- $(A5.1)\ M_2(e_2,m_1')\perp A$
- $(A5.2)\ M_2(e_2,m_1')\perp M_1|A$

$(A6)\ M_2(e_2,m_1')\perp M_1(e_1)$

DAG



$PIE_{effect27}$

$Definition$

$E\{[Y(1,1,1)-Y(1,1,0)-Y(1,0,1)+Y(1,0,0)-Y(0,1,1)+Y(0,1,0)+Y(0,0,1)-Y(0,0,0)]M_1(0)[M_2(1,1)-M_2(1,0)-M_2(0,1)+M_2(0,0)]*A\}$

$=\Big\{E\Big[Y\Big(A,M_1(0),M_2\big(A,M_1(0)\big)\Big)-Y\Big(A,M_1(0),M_2\big(0,M_1(0)\big)\Big)\Big]-E\big[Y\big(A,M_1(0),M_2(A,0)\big)-Y\big(A,M_1(0),M_2(0,0)\big)\big]$

$-E\Big[Y\Big(A,0,M_2\big(A,M_1(0)\big)\Big)-Y\Big(A,0,M_2\big(0,M_1(0)\big)\Big)\Big]+E\big[Y\big(A,0,M_2(A,0)\big)-Y\big(A,0,M_2(0,0)\big)\big]\Big\}$

$-\Big\{E\Big[Y\Big(0,M_1(0),M_2\big(A,M_1(0)\big)\Big)-Y\Big(0,M_1(0),M_2\big(0,M_1(0)\big)\Big)\Big]-E\big[Y\big(0,M_1(0),M_2(A,0)\big)-Y\big(0,M_1(0),M_2(0,0)\big)\big]$

$-E\Big[Y\Big(0,0,M_2\big(A,M_1(0)\big)\Big)-Y\Big(0,0,M_2\big(0,M_1(0)\big)\Big)\Big]+E\big[Y\big(0,0,M_2(A,0)\big)-Y\big(0,0,M_2(0,0)\big)\big]\Big\}$

$=\{[\phi_{33}(A,0,A,0)-\phi_{31}(A,0,0,0)]-[\phi_{27}(A,0,A,0)-\phi_{25}(A,0,0,0)]-[\phi_{17}(A,0,A,0)-\phi_{15}(A,0,0,0)]+[\phi_{10}(A,0,A,0)-\phi_{8}(A,0,0,0)]\}$

$-\{[\phi_{32}(0,0,A,0)-\phi_{6}(0,0,0,0)]-[\phi_{26}(0,0,A,0)-\phi_{5}(0,0,0,0)]-[\phi_{16}(0,0,A,0)-\phi_{3}(0,0,0,0)]+[\phi_{9}(0,0,A,0)-\phi_{2}(0,0,0,0)]\}$

$Identification$

$\phi_2=E\big[Y\big(a',m_1',M_2(e_2',m_1')\big)\big]=\Sigma_{m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')$

$\phi_3=E\Big[Y\Big(a',m_1',M_2\big(e_2',M_1(e_1')\big)\Big)\Big]=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=e_1')$

$\phi_5=E\big[Y\big(a',M_1(e_1'),M_2(e_2',m_1')\big)\big]=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(M_1=m_1|A=e_1')$

$\phi_6=E\Big[Y\Big(a',M_1(e_1'),M_2\big(e_2',M_1(e_1')\big)\Big)\Big]=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=e_1')$

$\phi_8=E\big[Y\big(A,m_1',M_2(e_2',m_1')\big)\big]=\Sigma_{a,m_2}E[Y|A=a,M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(A=a)$

$\phi_{10}=E\big[Y\big(A,m_1',M_2(A,m_1')\big)\big]=\Sigma_{a,m_2}E[Y|A=a,M_1=m_1',M_2=m_2]P(M_2=m_2|A=a,M_1=m_1')P(A=a)$

$\phi_{15}=E\Big[Y\Big(A,m_1',M_2\big(e_2',M_1(e_1')\big)\Big)\Big]=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=e_1')P(A=a)$

$\phi_{16}=E\Big[Y\Big(a',m_1',M_2\big(A,M_1(e_1')\big)\Big)\Big]=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=a,M_1=m_1)P(M_1=m_1|A=e_1')P(A=a)$

$\phi_{17}=E\Big[Y\Big(A,m_1',M_2\big(A,M_1(e_1')\big)\Big)\Big]=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1',M_2=m_2]P(M_2=m_2|A=a,M_1=m_1)P(M_1=m_1|A=e_1')P(A=a)$

$\phi_{25}=E\big[Y\big(A,M_1(e_1'),M_2(e_2',m_1')\big)\big]=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')P(M_1=m_1|A=e_1')P(A=a)$

$\phi_{26}=E\big[Y\big(a',M_1(e_1'),M_2(A,m_1')\big)\big]=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=a,M_1=m_1')P(M_1=m_1|A=e_1')P(A=a)$

$\phi_{27}=E\big[Y\big(A,M_1(e_1'),M_2(A,m_1')\big)\big]=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=a,M_1=m_1')P(M_1=m_1|A=e_1')P(A=a)$

$\phi_{32}=E\Big[Y\Big(a',M_1(e_1'),M_2\big(A,M_1(e_1')\big)\Big)\Big]=\Sigma_{a,m_1,m_2}E[Y|A=a',M_1=m_1,M_2=m_2]P(M_2=m_2|A=a,M_1=m_1)P(M_1=m_1|A=e_1')P(A=a)$

$\phi_{33}=E\Big[Y\Big(A,M_1(e_1'),M_2\big(A,M_1(e_1')\big)\Big)\Big]=\Sigma_{a,m_1,m_2}E[Y|A=a,M_1=m_1,M_2=m_2]P(M_2=m_2|A=a,M_1=m_1)P(M_1=m_1|A=e_1')P(A=a)$

$Required\ Assumptions$

$(A1)\ Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$

- $(A1.1)\ Y(a,m_1,m_2)\perp A$
- $(A1.2)\ Y(a,m_1,m_2)\perp M_1|A$
- $(A1.3)\ Y(a,m_1,m_2)\perp M_2|A,M_1$

$(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$

$(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$

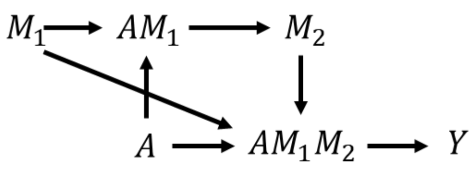
$(A4)\ M_1(e_1)\perp A$

$(A5)\ M_2(e_2,m_1')\perp\{A,M_1\}$

- $(A5.1)\ M_2(e_2,m_1')\perp A$
- $(A5.2)\ M_2(e_2,m_1')\perp M_1|A$

$(A6)\ M_2(e_2,m_1')\perp M_1(e_1)$

DAG



$PIE_{effect28}$

$Definition$

$$E\{[Y(1,1,1) - Y(1,1,0) - Y(1,0,1) + Y(1,0,0) - Y(0,1,1) + Y(0,1,0) + Y(0,0,1) - Y(0,0,0)]M_1(0)[M_2(A,0) - M_2(0,0)] * A\}$$

$$= E\big[Y\big(A, M_1(0), M_2(A,0)\big) - Y\big(0, M_1(0), M_2(A,0)\big)\big] - E\big[Y\big(A, 0, M_2(A,0)\big) - Y\big(A, 0, M_2(0,0)\big)\big] - E\big[Y\big(A, M_1(0), M_2(0,0)\big) - Y\big(0, M_1(0), M_2(0,0)\big)\big] + E\big[Y\big(0,0, M_2(A,0)\big) - Y\big(0,0, M_2(0,0)\big)\big]$$

$$= [\phi_{27}(A,0,A,0) - \phi_{26}(0,0,A,0)] - [\phi_{10}(A,0,A,0) - \phi_8(A,0,0,0)] - [\phi_{25}(A,0,0,0) - \phi_5(0,0,0,0)] + [\phi_9(0,0,A,0) - \phi_2(0,0,0,0)]$$

$Identification$

$\phi_2 = E\big[Y\big(a', m_1', M_2(e_2', m_1')\big)\big] = \Sigma_{m_2} E[Y|A = a', M_1 = m_1', M_2 = m_2]P(M_2 = m_2|A = e_2', M_1 = m_1')$

$\phi_5 = E\big[Y\big(a', M_1(e_1'), M_2(e_2', m_1')\big)\big] = \Sigma_{m_1,m_2} E[Y|A = a', M_1 = m_1, M_2 = m_2]P(M_2 = m_2|A = e_2', M_1 = m_1')P(M_1 = m_1|A = e_1')$

$\phi_8 = E\big[Y\big(A, m_1', M_2(e_2', m_1')\big)\big] = \Sigma_{a,m_2} E[Y|A = a, M_1 = m_1', M_2 = m_2]P(M_2 = m_2|A = e_2', M_1 = m_1')P(A = a)$

$\phi_9 = E\big[Y\big(a', m_1', M_2(A, m_1')\big)\big] = \Sigma_{a,m_2} E[Y|A = a', M_1 = m_1', M_2 = m_2]P(M_2 = m_2|A = a, M_1 = m_1')P(A = a)$

$\phi_{10} = E\big[Y\big(A, m_1', M_2(A, m_1')\big)\big] = \Sigma_{a,m_2} E[Y|A = a, M_1 = m_1', M_2 = m_2]P(M_2 = m_2|A = a, M_1 = m_1')P(A = a)$

$\phi_{25} = E\big[Y\big(A, M_1(e_1'), M_2(e_2', m_1')\big)\big] = \Sigma_{a,m_1,m_2} E[Y|A = a, M_1 = m_1, M_2 = m_2]P(M_2 = m_2|A = e_2', M_1 = m_1')P(M_1 = m_1|A = e_1')P(A = a)$

$\phi_{26} = E\big[Y\big(a', M_1(e_1'), M_2(A, m_1')\big)\big] = \Sigma_{a,m_1,m_2} E[Y|A = a', M_1 = m_1, M_2 = m_2]P(M_2 = m_2|A = a, M_1 = m_1')P(M_1 = m_1|A = e_1')P(A = a)$

$\phi_{27} = E\big[Y\big(A, M_1(e_1'), M_2(A, m_1')\big)\big] = \Sigma_{a,m_1,m_2} E[Y|A = a, M_1 = m_1, M_2 = m_2]P(M_2 = m_2|A = a, M_1 = m_1')P(M_1 = m_1|A = e_1')P(A = a)$

$Required\ Assumptions$

$(A1)$ $Y(a, m_1, m_2) \perp \{A, M_1, M_2\}$

- $(A1.1)$ $Y(a, m_1, m_2) \perp A$
- $(A1.2)$ $Y(a, m_1, m_2) \perp M_1|A$
- $(A1.3)$ $Y(a, m_1, m_2) \perp M_2|A, M_1$

$(A2)$ $Y(a, m_1, m_2) \perp M_1(e_1)$

$(A3)$ $Y(a, m_1, m_2) \perp M_2(e_2, m_1')$

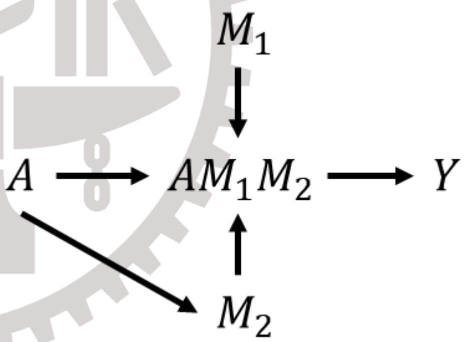
$(A4)$ $M_1(e_1) \perp A$

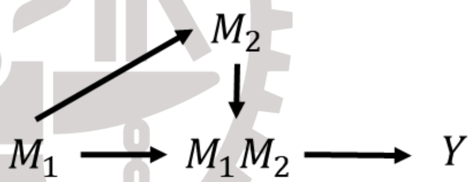
$(A5)$ $M_2(e_2, m_1') \perp \{A, M_1\}$

- $(A5.1)$ $M_2(e_2, m_1') \perp A$
- $(A5.2)$ $M_2(e_2, m_1') \perp M_1|A$

$(A6)$ $M_2(e_2, m_1') \perp M_1(e_1)$

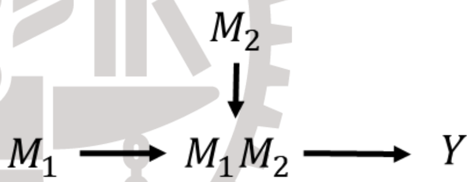
DAG



$PIE_{effect29}$	
$Definition$	
$E\{[Y(0,1,1) - Y(0,1,0) - Y(0,0,1) + Y(0,0,0)]M_1(0)[M_2(0,1) - M_2(0,0)]\}$ $= E\left[Y\left(0, M_1(0), M_2(0, M_1(0))\right) - Y(0, M_1(0), M_2(0,0))\right]$ $- E\left[Y\left(0,0, M_2(0, M_1(0))\right) - Y(0,0, M_2(0,0))\right]$ $= [\phi_6(0,0,0,0) - \phi_5(0,0,0,0)] - [\phi_3(0,0,0,0) - \phi_2(0,0,0,0)]$	
$Identification$	
$\phi_2 = E[Y(a', m_1', M_2(e_2', m_1'))]$ $= \Sigma_{m_2} E[Y\|A = a', M_1 = m_1', M_2 = m_2]P(M_2 = m_2\|A = e_2', M_1 = m_1')$ $\phi_3 = E\left[Y\left(a', m_1', M_2(e_2', M_1(e_3'))\right)\right]$ $= \Sigma_{m_1,m_2} E[Y\|A = a', M_1 = m_1', M_2 = m_2]P(M_2 = m_2\|A = e_2', M_1 = m_1)P(M_1 = m_1\|A = e_3')$ $\phi_5 = E[Y(a', M_1(e_1'), M_2(e_2', m_1'))]$ $= \Sigma_{m_1,m_2} E[Y\|A = a', M_1 = m_1, M_2 = m_2]P(M_2 = m_2\|A = e_2', M_1 = m_1')P(M_1 = m_1\|A = e_1')$ $\phi_6 = E\left[Y\left(a', M_1(e_1'), M_2(e_2', M_1(e_1'))\right)\right]$ $= \Sigma_{m_1,m_2} E[Y\|A = a', M_1 = m_1, M_2 = m_2]P(M_2 = m_2\|A = e_2', M_1 = m_1)P(M_1 = m_1\|A = e_1')$	
$Required\ Assumptions$	DAG
$(A1)\ Y(a, m_1, m_2) \perp \{A, M_1, M_2\}$ • $(A1.1)\ Y(a, m_1, m_2) \perp A$ • $(A1.2)\ Y(a, m_1, m_2) \perp M_1 \| A$ • $(A1.3)\ Y(a, m_1, m_2) \perp M_2 \| A, M_1$ $(A2)\ Y(a, m_1, m_2) \perp M_1(e_1)$ $(A3)\ Y(a, m_1, m_2) \perp M_2(e_2, m_1')$ $(A4)\ M_1(e_1) \perp A$ $(A5)\ M_2(e_2, m_1') \perp \{A, M_1\}$ • $(A5.1)\ M_2(e_2, m_1') \perp A$ • $(A5.2)\ M_2(e_2, m_1') \perp M_1 \| A$ $(A6)\ M_2(e_2, m_1') \perp M_1(e_1)$	

<table>
<tr><th colspan="2">$PIE_{effect30}$</th></tr>
<tr><td colspan="2">$Definition$</td></tr>
<tr><td colspan="2">$E\{[Y(0,1,1) - Y(0,1,0) - Y(0,0,1) + Y(0,0,0)]M_1(0)M_2(0,0)\}$
$= E\big[Y\big(0, M_1(0), M_2(0,0)\big) - Y\big(0,0, M_2(0,0)\big)\big] - E[Y(0, M_1(0), 0) - Y(0,0,0)]$
$= [\phi_5(0,0,0,0) - \phi_2(0,0,0,0)] - [\phi_4(0,0,0) - \phi_1(0,0,0)]$</td></tr>
<tr><td colspan="2">$Identification$</td></tr>
<tr><td colspan="2">$\phi_1 = E[Y(a', m_1', m_2')] = E[Y|A = a', M_1 = m_1', M_2 = m_2']$
$\phi_2 = E\big[Y\big(a', m_1', M_2(e_2', m_1')\big)\big]$
$= \Sigma_{m_2} E[Y|A = a', M_1 = m_1', M_2 = m_2]P(M_2 = m_2|A = e_2', M_1 = m_1')$
$\phi_4 = E[Y(a', M_1(e_1'), m_2')]$
$= \Sigma_{m_1} E[Y|A = a', M_1 = m_1, M_2 = m_2']P(M_1 = m_1|A = e_1')$
$\phi_5 = E\big[Y\big(a', M_1(e_1'), M_2(e_2', m_1')\big)\big]$
$= \Sigma_{m_1,m_2} E[Y|A = a', M_1 = m_1, M_2 = m_2]P(M_2 = m_2|A = e_2', M_1 = m_1')P(M_1 = m_1|A = e_1')$</td></tr>
<tr><td>$Required\ Assumptions$</td><td>DAG</td></tr>
<tr><td>$(A1)\ Y(a, m_1, m_2) \perp \{A, M_1, M_2\}$
● $(A1.1)\ Y(a, m_1, m_2) \perp A$
● $(A1.2)\ Y(a, m_1, m_2) \perp M_1|A$
● $(A1.3)\ Y(a, m_1, m_2) \perp M_2|A, M_1$

$(A2)\ Y(a, m_1, m_2) \perp M_1(e_1)$
$(A3)\ Y(a, m_1, m_2) \perp M_2(e_2, m_1')$
$(A4)\ M_1(e_1) \perp A$
$(A5)\ M_2(e_2, m_1') \perp \{A, M_1\}$
● $(A5.1)\ M_2(e_2, m_1') \perp A$
● $(A5.2)\ M_2(e_2, m_1') \perp M_1|A$

$(A6)\ M_2(e_2, m_1') \perp M_1(e_1)$</td><td>
</td></tr>
</table>

<table>
<tr><th colspan="2">$PIE_{effect31}$</th></tr>
<tr><td colspan="2">$Definition$</td></tr>
<tr><td colspan="2">$E\{[Y(0,0,1)-Y(0,0,0)]M_1(0)[M_2(0,1)-M_2(0,0)]\}$
$=E\left[Y\left(0,0,M_2\big(0,M_1(0)\big)\right)-Y\big(0,0,M_2(0,0)\big)\right]$
$=[\phi_3(0,0,0,0)-\phi_2(0,0,0,0)]$</td></tr>
<tr><td colspan="2">$Identification$</td></tr>
<tr><td colspan="2">$\phi_2=E\big[Y\big(a',m_1',M_2(e_2',m_1')\big)\big]$
$=\Sigma_{m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1')$
$\phi_3=E\left[Y\left(a',m_1',M_2\big(e_2',M_1(e_1')\big)\right)\right]$
$=\Sigma_{m_1,m_2}E[Y|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2|A=e_2',M_1=m_1)P(M_1=m_1|A=e_1')$</td></tr>
<tr><td>$Required\ Assumptions$</td><td>DAG</td></tr>
<tr><td>$(A1)\ Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$
● $(A1.1)\ Y(a,m_1,m_2)\perp A$
● $(A1.2)\ Y(a,m_1,m_2)\perp M_1|A$
● $(A1.3)\ Y(a,m_1,m_2)\perp M_2|A,M_1$

$(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$
$(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$
$(A4)\ M_1(e_1)\perp A$
$(A5)\ M_2(e_2,m_1)\perp\{A,M_1\}$
● $(A5.1)\ M_2(e_2,m_1)\perp A$
● $(A5.2)\ M_2(e_2,m_1)\perp M_1|A$

$(A6)\ M_2(e_2,m_1')\perp M_1(e_1)$</td><td></td></tr>
</table>

<table>
<tr><th colspan="2">$PIE_{effect32}$</th></tr>
<tr><td colspan="2">$Definition$</td></tr>
<tr><td colspan="2">$E\{[Y(0,1,0)-Y(0,0,0)]M_1(0)\}$
$=E[Y(0,M_1(0),0)-Y(0,0,0)]$
$=[\phi_4(0,0,0)-\phi_1(0,0,0)]$</td></tr>
<tr><td colspan="2">$Identification$</td></tr>
<tr><td colspan="2">$\phi_1=E[Y(a',m_1',m_2')]=E[Y\|A=a',M_1=m_1',M_2=m_2']$
$\phi_4=E[Y(a',M_1(e_1'),m_2')]$
$=\Sigma_{m_1}E[Y\|A=a',M_1=m_1,M_2=m_2']P(M_1=m_1\|A=e_1')$</td></tr>
<tr><td>$Required\ Assumptions$</td><td>DAG</td></tr>
<tr><td>$(A1)\ Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$
● $(A1.1)\ Y(a,m_1,m_2)\perp A$
● $(A1.2)\ Y(a,m_1,m_2)\perp M_1\|A$
● $(A1.3)\ Y(a,m_1,m_2)\perp M_2\|A,M_1$
$(A2)\ Y(a,m_1,m_2)\perp M_1(e_1)$
$(A4)\ M_1(e_1)\perp A$</td><td>
</td></tr>
</table>

<table>
<tr><th colspan="2">$PIE_{effect33}$</th></tr>
<tr><td colspan="2">$Definition$</td></tr>
<tr><td colspan="2">$E\{[Y(0,0,1)-Y(0,0,0)]M_2(0,0)\}$
$=E\left[Y\left(0,0,M_2(0,0)\right)-Y(0,0,0)\right]$
$=[\phi_2(0,0,0,0)-\phi_1(0,0,0)]$</td></tr>
<tr><td colspan="2">$Identification$</td></tr>
<tr><td colspan="2">$\phi_1=E[Y(a',m_1',m_2')]=E[Y\|A=a',M_1=m_1',M_2=m_2']$
$\phi_2=E\left[Y\left(a',m_1',M_2(e_2',m_1')\right)\right]=\Sigma_{m_2}E[Y\|A=a',M_1=m_1',M_2=m_2]P(M_2=m_2\|A=e_2',M_1=m_1')$</td></tr>
<tr><td>$Required\ Assumptions$</td><td>DAG</td></tr>
<tr><td>$(A1)\ Y(a,m_1,m_2)\perp\{A,M_1,M_2\}$
● $(A1.1)\ Y(a,m_1,m_2)\perp A$
● $(A1.2)\ Y(a,m_1,m_2)\perp M_1\|A$
● $(A1.3)\ Y(a,m_1,m_2)\perp M_2\|A,M_1$
$(A3)\ Y(a,m_1,m_2)\perp M_2(e_2,m_1')$
$(A5)\ M_2(e_2,m_1)\perp\{A,M_1\}$
● $(A5.1)\ M_2(e_2,m_1)\perp A$
● $(A5.2)\ M_2(e_2,m_1)\perp M_1\|A$</td><td>
</td></tr>
</table>

3.4 三十三路徑 PIE 拆解法與過往拆解法的比較

於 3.1 節我們介紹了當前學者(Sjölander, 2018)、(Fulcher et al. ,2020)、(O′Connell et al. ,2022)以及本研究室(Duan, 2024)提出的 PIE、PAF 尺度的拆解法，並說明兩個尺度的關係為$PAF = \frac{PIE}{E[Y]}$。

本節將介紹上述 PIE 三十三路徑拆解法與過往拆解法的關係，為了便於在同一尺度下討論，我們主要聚焦於 PIE 尺度，並且對於(Sjölander, 2018) 提出的 PAF 尺度命名方式進行調整，以更清晰地識別我們所討論的效應。具體而言，學者(Sjölander, 2018) 提出的 NIAF(natural indirect attributable fraction) 與 NDAF(natural direct attributable fraction)，將分別稱重新命名成 NIIE(natural indirect intervention effect)、NDIE(natural direct intervention effect)，以符合 PIE 尺度的討論架構。

此外，在(O′Connell et al. ,2022)提出的多重中介因子 PAF 拆解法中，我們主要關注其機制型 PS-PAF 的關係，由於機制型 PS-PAF 主要是將(Sjölander, 2018) 的二路徑拆解法拓展至雙平行中介因子，與我們接下來討論的雙中介因子比較的情況相符，因此，我們主要比較(Sjölander, 2018)、(Fulcher et al. ,2020) 以及本研究室(Duan, 2024)提出的拆解法。下圖主要呈現雙平行中介因子與雙有序中介因子之因果圖，可以協助我們理解平行和有序中介因子的差異。

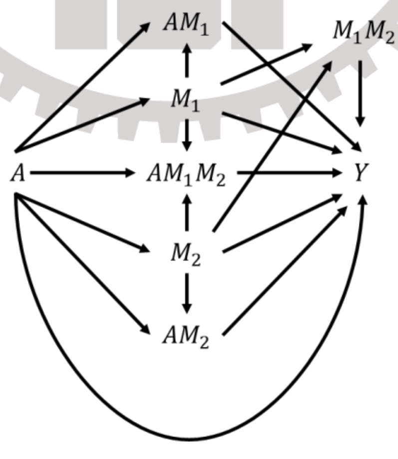


圖 3.3 雙平行中介因子中介效應與交互作用的因果圖

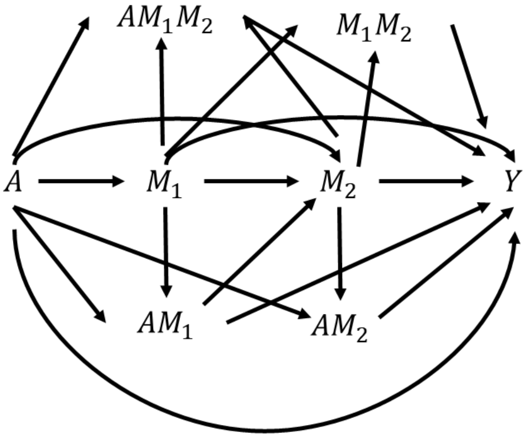


圖 3.4 雙有序中介因子中介效應與交互作用的因果圖

從直觀的因果圖結構中我們可以發現,雙平行中介因子與雙有序中介因子的差別主要在於，平行中介因子不包含中介因子M_1指向中介因子M_2的因果效應。因此，在雙平行中介因子的情境下，效應總數相對較少，原先的三十三個族群介入效應會減少至十七個族群介入效應。當只考慮M_1為中介因子的時候，$PIE_{effect33}$於拆解過程中會被消除,表示沒有來自中介因子M_2進而影響結果變量Y的效應。這一差異強調了在不同的中介因子結構下，PIE 拆解法對於因果效應的解析方式。下表 3.11、3.12 我們將呈現分別呈現雙平行、有序中介因子考慮M_1為中介因子以及同時考慮M_1、M_2為中介因子之過往拆解法的比較。

表 3.11 雙平行中介因子過往拆解法的比較

雙平行中介因子		$As\ M_1\ Mediator$			$As\ M_1\&M_2\ Mediator$		
	PIE 33way	(Sjölander)	(Fulcher, et al)	(Duan)	(Sjölander)	(Fulcher, et al.)	(Duan)
	$PIE_{effect1}$	$NDIE^{M_1}$	$PIDE^{M_1}$	$PIE_{CDE}^{M_1}$	$NDIE^{M_1M_2}$	$PIDE^{M_1M_2}$	$PIE_{CDE}^{M_1M_2}$
	$PIE_{effect2}$			$PIE_{INT_{ref}}^{M_1}$			$PIE_{INT_{ref}}^{M_1M_2}$
	$PIE_{effect3}$			$PIE_{CDE}^{M_1}$			
	$PIE_{effect4}$			$PIE_{INT_{ref}}^{M_1}$			
	$PIE_{effect5}$						
	$PIE_{effect6}$						
	$PIE_{effect7}$	$NIIE^{M_1}$	$PIIE^{M_1}$	$PIE_{PIE}^{M_1}$	$NIIE^{M_1M_2}$	$PIIE^{M_1M_2}$	$PIE_{PIE}^{M_1M_2}$
	$PIE_{effect8}$	$NDIE^{M_1}$	$PIDE^{M_1}$	$PIE_{CDE}^{M_1}$			
	$PIE_{effect9}$						
	$PIE_{effect10}$						
	$PIE_{effect11}$	$NIIE^{M_1}$	$PIIE^{M_1}$	$PIE_{PIE}^{M_1}$	$NIIE^{M_1M_2}$	$PIIE^{M_1M_2}$	$PIE_{PIE}^{M_1M_2}$
	$PIE_{effect12}$						
	$PIE_{effect13}$						
	$PIE_{effect14}$	$NDIE^{M_1}$	$PIIE^{M_1}$	$PIE_{INT_{med}}^{M_1}$	$NIIE^{M_1M_2}$	$PIIE^{M_1M_2}$	$PIE_{PIE}^{M_1M_2}$
	$PIE_{effect15}$						
	$PIE_{effect16}$	$NDIE^{M_1}$	$PIDE^{M_1}$	$PIE_{INT_{ref}}^{M_1}$	$NIIE^{M_1M_2}$	$PIIE^{M_1M_2}$	$PIE_{PIE}^{M_1M_2}$
	$PIE_{effect17}$						
	$PIE_{effect18}$	$NDIE^{M_1}$	$PIIE^{M_1}$	$PIE_{INT_{med}}^{M_1}$	$NDIE^{M_1M_2}$	$PIIE^{M_1M_2}$	$PIE_{INT_{med}}^{M_1M_2}$
	$PIE_{effect19}$		$PIDE^{M_1}$	$PIE_{CDE}^{M_1}$			
	$PIE_{effect20}$						
	$PIE_{effect21}$	$NDIE^{M_1}$	$PIIE^{M_1}$	$PIE_{INT_{med}}^{M_1}$	$NDIE^{M_1M_2}$	$PIIE^{M_1M_2}$	$PIE_{INT_{med}}^{M_1M_2}$
	$PIE_{effect22}$						
	$PIE_{effect23}$						
	$PIE_{effect24}$						
	$PIE_{effect25}$	$NDIE^{M_1}$	$PIIE^{M_1}$	$PIE_{INT_{med}}^{M_1}$	$NDIE^{M_1M_2}$	$PIIE^{M_1M_2}$	$PIE_{INT_{med}}^{M_1M_2}$
	$PIE_{effect26}$						
	$PIE_{effect27}$						
	$PIE_{effect28}$	$NDIE^{M_1}$	$PIDE^{M_1}$	$PIE_{INT_{ref}}^{M_1}$	$NDIE^{M_1M_2}$	$PIIE^{M_1M_2}$	$PIE_{INT_{med}}^{M_1M_2}$
	$PIE_{effect29}$						
	$PIE_{effect30}$	$NIIE^{M_1}$	$PIDE^{M_1}$	$PIE_{CEM}^{M_1}$	$NIIE^{M_1M_2}$	$PIDE^{M_1M_2}$	$PIE_{CEM}^{M_1M_2}$
	$PIE_{effect31}$						
	$PIE_{effect32}$	$NIIE^{M_1}$	$PIDE^{M_1}$	$PIE_{CEM}^{M_1}$	$NIIE^{M_1M_2}$	$PIDE^{M_1M_2}$	$PIE_{CEM}^{M_1M_2}$
	$PIE_{effect33}$						$PIE_{CEM}^{M_1M_2}$

表 3.12 雙有序中介因子過往拆解法的比較

<table>
<tr><td colspan="2">雙有序中介因子</td><td colspan="3">$As\ M_1\ Mediator$</td><td colspan="3">$As\ M_1\&M_2\ Mediator$</td></tr>
<tr><td></td><td>PIE 33way</td><td>(Sjölander)</td><td>(Fulcher, et al)</td><td>(Duan)</td><td>(Sjölander)</td><td>(Fulcher, et al.)</td><td>(Duan)</td></tr>
<tr><td rowspan="33"></td><td>$PIE_{effect1}$</td><td rowspan="6">$NDIE^{M_1}$</td><td rowspan="6">$PIDE^{M_1}$</td><td>$PIE_{CDE}^{M_1}$</td><td rowspan="6">$NDIE^{M_1M_2}$</td><td rowspan="6">$PIDE^{M_1M_2}$</td><td>$PIE_{CDE}^{M_1M_2}$</td></tr>
<tr><td>$PIE_{effect2}$</td><td>$PIE_{INT_{ref}}^{M_1}$</td><td rowspan="5">$PIE_{INT_{ref}}^{M_1M_2}$</td></tr>
<tr><td>$PIE_{effect3}$</td><td>$PIE_{CDE}^{M_1}$</td></tr>
<tr><td>$PIE_{effect4}$</td><td rowspan="3">$PIE_{INT_{ref}}^{M_1}$</td></tr>
<tr><td>$PIE_{effect5}$</td></tr>
<tr><td>$PIE_{effect6}$</td></tr>
<tr><td>$PIE_{effect7}$</td><td>$NIIE^{M_1}$</td><td>$PIIE^{M_1}$</td><td>$PIE_{PIE}^{M_1}$</td><td rowspan="11">$NIIE^{M_1M_2}$</td><td rowspan="22">$PIIE^{M_1M_2}$</td><td rowspan="11">$PIE_{PIE}^{M_1M_2}$</td></tr>
<tr><td>$PIE_{effect8}$</td><td>$NDIE^{M_1}$</td><td>$PIDE^{M_1}$</td><td>$PIE_{CDE}^{M_1}$</td></tr>
<tr><td>$PIE_{effect9}$</td><td rowspan="3">$NIIE^{M_1}$</td><td rowspan="3">$PIIE^{M_1}$</td><td rowspan="3">$PIE_{PIE}^{M_1}$</td></tr>
<tr><td>$PIE_{effect10}$</td></tr>
<tr><td>$PIE_{effect11}$</td></tr>
<tr><td>$PIE_{effect12}$</td><td rowspan="17">$NDIE^{M_1}$</td><td>$PIDE^{M_1}$</td><td>$PIE_{INT_{ref}}^{M_1}$</td></tr>
<tr><td>$PIE_{effect13}$</td><td rowspan="2">$PIIE^{M_1}$</td><td rowspan="2">$PIE_{INT_{med}}^{M_1}$</td></tr>
<tr><td>$PIE_{effect14}$</td></tr>
<tr><td>$PIE_{effect15}$</td><td rowspan="2">$PIDE^{M_1}$</td><td rowspan="2">$PIE_{INT_{ref}}^{M_1}$</td></tr>
<tr><td>$PIE_{effect16}$</td></tr>
<tr><td>$PIE_{effect17}$</td><td rowspan="2">$PIIE^{M_1}$</td><td rowspan="2">$PIE_{INT_{med}}^{M_1}$</td></tr>
<tr><td>$PIE_{effect18}$</td><td rowspan="11">$NDIE^{M_1M_2}$</td><td rowspan="11">$PIE_{INT_{med}}^{M_1M_2}$</td></tr>
<tr><td>$PIE_{effect19}$</td><td>$PIDE^{M_1}$</td><td>$PIE_{CDE}^{M_1}$</td></tr>
<tr><td>$PIE_{effect20}$</td><td rowspan="4">$PIIE^{M_1}$</td><td rowspan="4">$PIE_{INT_{med}}^{M_1}$</td></tr>
<tr><td>$PIE_{effect21}$</td></tr>
<tr><td>$PIE_{effect22}$</td></tr>
<tr><td>$PIE_{effect23}$</td></tr>
<tr><td>$PIE_{effect24}$</td><td>$PIDE^{M_1}$</td><td>$PIE_{INT_{ref}}^{M_1}$</td></tr>
<tr><td>$PIE_{effect25}$</td><td rowspan="2">$PIIE^{M_1}$</td><td rowspan="2">$PIE_{INT_{med}}^{M_1}$</td></tr>
<tr><td>$PIE_{effect26}$</td></tr>
<tr><td>$PIE_{effect27}$</td><td rowspan="6">$PIDE^{M_1}$</td><td rowspan="2">$PIE_{INT_{ref}}^{M_1}$</td></tr>
<tr><td>$PIE_{effect28}$</td></tr>
<tr><td>$PIE_{effect29}$</td><td rowspan="4">$NIIE^{M_1}$</td><td rowspan="4">$PIE_{CEM}^{M_1}$</td><td rowspan="4">$NIIE^{M_1M_2}$</td><td rowspan="5">$PIDE^{M_1M_2}$</td><td rowspan="5">$PIE_{CEM}^{M_1M_2}$</td></tr>
<tr><td>$PIE_{effect30}$</td></tr>
<tr><td>$PIE_{effect31}$</td></tr>
<tr><td>$PIE_{effect32}$</td></tr>
<tr><td>$PIE_{effect33}$</td><td colspan="3"></td><td>$NDIE^{M_1M_2}$</td></tr>
</table>

由上表可以發現，無論是在雙有序中介因子或雙平行中介因子，我們所提出的三十三路徑 PIE 拆解法能夠最細緻的表達所有可能的中介效應及交互作用效應，可見本研究提出之拆解法能夠更準確的歸類各效應背後的因果機制，進而做出更準確的解釋與結論。

第四章、結論

本研究回顧雙有序中介因子之中介效應與交互作用的二十八路徑拆解法，並且透過因果參數ϕ_1至ϕ_6的反事實模型，以及透過合理的辨識假設，得出二十八路徑效應的辨識結果，完整改進二十八路徑拆解法在因果推論的分析架構，並且將雙有序中介因子二十八路徑拆解法拓展至族群介入效應(PIE)的尺度，發現其因果參數會增加至三十三個，但是都為ϕ_1至ϕ_6的延伸因果參數，差異在於需要考量暴露因子A於現實世界的觀測值，最後呈現其辨識假設與辨識結果，並比較過去PIE 尺度下的拆解法與雙有序中介因子三十三路徑拆解法的差異，期望為未來的公共衛生研究的統計指標，提供完整的因果推論架構。

不過本研究上有可以改進之處。首先，本研究僅在雙有序中介因子下進行最細部拆解，可以在未來延伸至 K 個中介因子，使拆解方法更具通用性，但是可能會拆解出更多複雜的效應，其實用性仍需更多的研究協助；其次是本研究雖然有完整的因果分析架構，不過在雙有序中介因子平均因果效應的尺度上，還未與其他學者的拆解法進行比較，可以在未來與其他學者的拆解法進行比較，以證實二十八路徑拆解法在因果推論的分析框架上是更加細緻的；第三、雖然在因果推論的架構上本文已經提供了完整的辨識結果，但仍需要實際的資料分析，提供因果方法學的供因果方法學的應用實證，進一步驗證二十八路徑拆解法在實務上的可行性與準確性。

總結而言，本研究對雙有序中介因子之二十八路徑拆解法進行了完整的因果推論分析，並將其拓展至族群介入效應的尺度。然而，未來仍需進一步的研究來驗證本方法的適用性與實用性，並與現有的因果推論方法進行比較，為公共衛生與流行病學等領域的因果推論提供更細緻的分析工具。

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第六章、附錄

6.1 雙有序中介因子 PIE 拆解過程

$E[Y-Y(0,0,0)]$

$$= E\left[Y - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right) + Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right) - Y(0,0,0)\right]$$

$$= E\left[Y - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right) + Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right) - Y(0,0,0)\right]$$
$$+ E\left[Y\left(A, M_1(0), M_2\left(0, M_1(0)\right)\right) - Y\left(A, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$$= E\left[Y\left(A, M_1(0), M_2\left(0, M_1(0)\right)\right) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$
$$+ E\left[Y - Y\left(A, M_1(0), M_2\left(0, M_1(0)\right)\right) + Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right.$$
$$\left. - Y(0,0,0)\right] + E\left[Y\left(0, M_1, M_2(0, M_1)\right) - Y\left(0, M_1, M_2(0, M_1)\right)\right]$$
$$+ E\left[Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$$= E\left[Y\left(A, M_1(0), M_2\left(0, M_1(0)\right)\right) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$
$$+ E\left[Y\left(0, M_1, M_2(0, M_1)\right) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$
$$+ E\left[Y - Y\left(A, M_1(0), M_2\left(0, M_1(0)\right)\right) + Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right.$$
$$\left. - Y(0,0,0)\right] - E\left[Y\left(0, M_1, M_2(0, M_1)\right)\right]$$
$$+ E\left[Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$
$$+ E\left[Y\left(0, M_1(0) M_2\left(A, M_1(0)\right)\right) - Y\left(0, M_1(0), M_2\left(A, M_1(0)\right)\right)\right]$$
$$+ E\left[Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$$= E\left[Y\left(A, M_1(0), M_2\left(0, M_1(0)\right)\right) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$$+ E\left[Y\left(0, M_1, M_2(0, M_1)\right) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$$+ E\left[Y\left(0, M_1(0), M_2\left(A, M_1(0)\right)\right) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$$+ E\left[Y - Y\left(A, M_1(0), M_2\left(0, M_1(0)\right)\right) + Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right.$$

$$\left. - Y(0,0,0)\right] - E\left[Y\left(0, M_1, M_2(0, M_1)\right)\right]$$

$$+ E\left[Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right] - E\left[Y\left(0, M_1(0), M_2\left(A, M_1(0)\right)\right)\right]$$

$$+ E\left[Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$$+ E\left[Y\left(A, M_1, M_2(0, M_1)\right) - Y\left(A, M_1, M_2(0, M_1)\right)\right]$$

$$= E\left[Y\left(A, M_1(0), M_2\left(0, M_1(0)\right)\right) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$$+ E\left[Y\left(0, M_1, M_2(0, M_1)\right) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$$+ E\left[Y\left(0, M_1(0), M_2\left(A, M_1(0)\right)\right) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$$+ E\left[Y\left(A, M_1, M_2(0, M_1)\right) - Y\left(A, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$$- E\left[Y\left(0, M_1, M_2(0, M_1)\right) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$$+ E\left[Y - Y(0,0,0) + Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right.$$

$$\left. - Y\left(0, M_1(0), M_2\left(A, M_1(0)\right)\right)\right]$$

$$+ E\left[Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right) - Y\left(A, M_1, M_2(0, M_1)\right)\right]$$

$$+ E\left[Y\left(A, M_1(0), M_2\left(A, M_1(0)\right)\right) - Y\left(A, M_1(0), M_2\left(A, M_1(0)\right)\right)\right]$$

$$+ E\left[Y\left(A, M_1(0), M_2\left(0, M_1(0)\right)\right) - Y\left(A, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$$= E\left[Y\left(A, M_1(0), M_2\left(0, M_1(0)\right)\right) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$$+ E\left[Y(0, M_1, M_2(0, M_1)) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$$+ E\left[Y\left(0, M_1(0), M_2\left(A, M_1(0)\right)\right) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$$+ E\left[Y(A, M_1, M_2(0, M_1)) - Y\left(A, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$$- E\left[Y(0, M_1, M_2(0, M_1)) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$$+ E\left[Y\left(A, M_1(0), M_2\left(A, M_1(0)\right)\right) - Y\left(A, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$$- E\left[Y\left(0, M_1(0), M_2\left(A, M_1(0)\right)\right) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$$+ E\left[Y - Y(0,0,0) + Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right.$$

$$\left. - Y\left(A, M_1, M_2(0, M_1)\right)\right] - E\left[Y\left(A, M_1(0), M_2\left(A, M_1(0)\right)\right)\right]$$

$$+ E\left[Y\left(A, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$$+ E[Y(0, M_1, M_2(A, M_1)) - Y(0, M_1, M_2(A, M_1))]$$

$$+ E[Y(0, M_1, M_2(0, M_1)) - Y(0, M_1, M_2(0, M_1))]$$

$$+ E\left[Y\left(0, M_1(0), M_2\left(A, M_1(0)\right)\right) - Y\left(0, M_1(0), M_2\left(A, M_1(0)\right)\right)\right]$$

$$+ E\left[Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$$= E\left[Y\left(A, M_1(0), M_2\left(0, M_1(0)\right)\right) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right] (PSE_0)$$

$$+E\left[Y(0, M_1, M_2(0, M_1)) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right] (PSE_1)$$

$$+E\left[Y\left(0, M_1(0), M_2\left(A, M_1(0)\right)\right) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right] (PSE_2)$$

$$+E\left[Y(A, M_1, M_2(0, M_1)) - Y\left(A, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$$- E\left[Y(0, M_1, M_2(0, M_1)) - Y\left(0, M_1(0), M_2\left(0, M_1(0)\right)\right)\right]$$

$\left(INT_{med}(A - M_1)\right)$

$$+E\left[Y\left(A, M_1(0), M_2\big(A, M_1(0)\big)\right) - Y\left(A, M_1(0), M_2\big(0, M_1(0)\big)\right)\right]$$

$$- E\left[Y\left(0, M_1(0), M_2\big(A, M_1(0)\big)\right) - Y\left(0, M_1(0), M_2\big(0, M_1(0)\big)\right)\right]$$

$\left(INT_{med}(A - M_2)\right)$

$$+E\left[Y\big(0, M_1, M_2(A, M_1)\big) - Y\big(0, M_1, M_2(0, M_1)\big)\right]$$

$$- E\left[Y\left(0, M_1(0), M_2\big(A, M_1(0)\big)\right) - Y\left(0, M_1(0), M_2\big(0, M_1(0)\big)\right)\right]$$

$\left(INT_{med}(M_1 - M_2)\right)$

$$+\Big\{E\left[Y - Y\big(A, M_1, M_2(0, M_1)\big)\right]$$

$$- E\left[Y\left(A, M_1(0), M_2\big(A, M_1(0)\big)\right) - Y\left(A, M_1(0), M_2\big(0, M_1(0)\big)\right)\right]\Big\}$$

$$-\Big\{E\left[Y\big(0, M_1, M_2(A, M_1)\big) - Y\big(0, M_1, M_2(0, M_1)\big)\right]$$

$$- E\left[Y\left(0, M_1(0), M_2\big(A, M_1(0)\big)\right) - Y\left(0, M_1(0), M_2\big(0, M_1(0)\big)\right)\right]\Big\}$$

$\left(INT_{med}(A - M_1 - M_2)\right)$

$$+E\left[Y\left(0, M_1(0), M_2\big(0, M_1(0)\big)\right) - Y(0,0,0)\right]\ (CEM)$$

$\boldsymbol{PSE_0}$

$$E\left[Y\left(A, M_1(0), M_2\big(0, M_1(0)\big)\right) - Y\left(0, M_1(0), M_2\big(0, M_1(0)\big)\right)\right]$$

$$= E[Y(A, 0,0) - Y(0,0,0)]\ (PIE_{effect1})$$

$$+E[Y(A, M_1(0), 0) - Y(0, M_1(0), 0)] - E[Y(A, 0,0) - Y(0,0,0)](PIE_{effect2})$$

$$+E\big[Y\big(A, 0, M_2(0,0)\big) - Y\big(0,0, M_2(0,0)\big)\big] - E[Y(A, 0,0) - Y(0,0,0)](PIE_{effect3})$$

$$\begin{aligned}+E\big[Y\big(A. M_1(0), M_2(0,0)\big) - Y\big(0, M_1(0), M_2(0,0)\big)\big]& \\ - E[Y(A, M_1(0), 0) - Y(0, M_1(0), 0)]& \\ - E\big[Y\big(A, 0, M_2(0,0)\big) - Y\big(0,0, M_2(0,0)\big)\big]& \\ + E[Y(A, 0,0) - Y(0,0,0)]\ (PIE_{effect4})&\end{aligned}$$

$$\begin{aligned}+E\left[Y\left(A, M_1(0), M_2\big(0, M_1(0)\big)\right) - Y\left(0, M_1(0), M_2\big(0, M_1(0)\big)\right)\right]& \\ - E\big[Y\big(A, M_1(0), M_2(0,0)\big) - Y\big(0, M_1(0), M_2(0,0)\big)\big]& \\ - E\left[Y\left(A, 0, M_2\big(0, M_1(0)\big)\right) - Y\left(0,0, M_2\big(0, M_1(0)\big)\right)\right]& \\ + E\big[Y\big(A, 0, M_2(0,0)\big) - Y\big(0,0, M_2(0,0)\big)\big](PIE_{effect5})&\end{aligned}$$

$$\begin{aligned}+E\left[Y\left(A, 0, M_2\big(0, M_1(0)\big)\right) - Y\left(0,0, M_2\big(0, M_1(0)\big)\right)\right]& \\ - E\big[Y\big(A, 0, M_2(0,0)\big) - Y\big(0,0, M_2(0,0)\big)\big](PIE_{effect6})&\end{aligned}$$

$\boldsymbol{PSE_1}$

$$E\left[Y\big(0, M_1, M_2(0, M_1)\big) - Y\left(0, M_1(0), M_2\big(0, M_1(0)\big)\right)\right]$$

$$= E[Y(0, M_1, 0) - Y(0, M_1(0), 0)]\ (PIE_{effect7})$$

$$+E\left[Y\big(0, M_1, M_2(0, M_1)\big) - Y\left(0, M_1(0), M_2\big(0, M_1(0)\big)\right)\right]$$
$$- E\big[Y\big(0, M_1, M_2(0,0)\big) - Y\big(0, M_1(0), M_2(0,0)\big)\big]$$
$$- E\left[Y\big(0,0, M_2(0, M_1)\big) - Y\left(0,0, M_2\big(0, M_1(0)\big)\right)\right] (PIE_{effect9})$$

$$+E\left[Y\big(0,0, M_2(0, M_1)\big) - Y\left(0,0, M_2\big(0, M_1(0)\big)\right)\right] (PIE_{effect10})$$

$$+E\big[Y\big(0, M_1, M_2(0,0)\big) - Y\big(0, M_1(0), M_2(0,0)\big)\big]$$
$$- E[Y(0, M_1, 0) - Y(0, M_1(0), 0)](PIE_{effect11})$$

$\boldsymbol{PSE_2}$

$$E\left[Y\left(0, M_1(0), M_2\big(A, M_1(0)\big)\right) - Y\left(0, M_1(0), M_2\big(0, M_1(0)\big)\right)\right]$$

$$= E\big[Y\big(0,0, M_2(A, 0)\big) - Y\big(0,0, M_2(0,0)\big)\big](PIE_{effect8})$$

$$+E\left[Y\left(0,0, M_2\big(A, M_1(0)\big)\right) - Y\left(0,0, M_2\big(0, M_1(0)\big)\right)\right]$$
$$- E\big[Y\big(0,0, M_2(A, 0)\big) - Y\big(0,0, M_2(0,0)\big)\big](PIE_{effect12})$$

$$+E\left[Y\left(0, M_1(0), M_2\big(A, M_1(0)\big)\right) - Y\left(0, M_1(0), M_2\big(0, M_1(0)\big)\right)\right]$$
$$- E\big[Y\big(0, M_1(0), M_2(A, 0)\big) - Y\big(0, M_1(0), M_2(0,0)\big)\big]$$
$$- E\left[Y\left(0,0, M_2\big(A, M_1(0)\big)\right) - Y\left(0,0, M_2\big(0, M_1(0)\big)\right)\right]$$
$$- E\big[Y\big(0,0, M_2(A, 0)\big) - Y\big(0,0, M_2(0,0)\big)\big]\ (PIE_{effect15})$$

$$+E\big[Y\big(0, M_1(0), M_2(A, 0)\big) - Y\big(0, M_1(0), M_2(0,0)\big)\big]$$
$$-E\big[Y\big(0,0, M_2(A, 0)\big) - Y\big(0,0, M_2(0,0)\big)\big]\ (PIE_{effect16})$$

$\boldsymbol{INT_{med}(A-M_1)}$

$$E\left[Y\big(A,M_1,M_2(0,M_1)\big)-Y\left(A,M_1(0),M_2\big(0,M_1(0)\big)\right)\right]$$

$$-E\left[Y\big(0,M_1,M_2(0,M_1)\big)-Y\left(0,M_1(0),M_2\big(0,M_1(0)\big)\right)\right]$$

$$=E[Y(A,M_1,0)-Y(0,M_1,0)]-E[Y(A,M_1(0),0)-Y(0,M_1(0),0)]\ (PIE_{effect18})$$

$$+E\left[Y\big(A,0,M_2(0,M_1)\big)-Y\left(A,0,M_2\big(0,M_1(0)\big)\right)\right]$$

$$-E\left[Y\big(0,0,M_2(0,M_1)\big)-Y\left(0,0,M_2\big(0,M_1(0)\big)\right)\right]\ (PIE_{effect20})$$

$$\begin{aligned}+E\big[Y\big(A,M_1,M_2(0,0)\big)-Y\big(A,M_1(0),M_2(0,0)\big)\big]\\ -E\big[Y\big(0,M_1,M_2(0,0)\big)-Y\big(0,M_1(0),M_2(0,0)\big)\big]\\ -E[Y(A,M_1,0)-Y(0,M_1,0)]\\ +E[Y(A,M_1(0),0)-Y(0,M_1(0),0)](PIE_{effect21})\end{aligned}$$

$$+\Big\{E\left[Y\big(A,M_1,M_2(0,M_1)\big)-Y\left(A,M_1(0),M_2\big(0,M_1(0)\big)\right)\right]$$

$$-E\big[Y\big(A,M_1,M_2(0,0)\big)-Y\big(A,M_1(0),M_2(0,0)\big)\big]$$

$$-E\left[Y\big(A,0,M_2(0,M_1)\big)-Y\left(A,0,M_2\big(0,M_1(0)\big)\right)\right]\Big\}$$

$$-\Big\{E\left[Y\big(0,M_1,M_2(0,M_1)\big)-Y\left(0,M_1(0),M_2\big(0,M_1(0)\big)\right)\right]$$

$$-E\big[Y\big(0,M_1,M_2(0,0)\big)-Y\big(0,M_1(0),M_2(0,0)\big)\big]$$

$$-E\left[Y\big(0,0,M_2(0,M_1)\big)-Y\left(0,0,M_2\big(0,M_1(0)\big)\right)\right]\Big\}\ (PIE_{effect22})$$

$\boldsymbol{INT_{med}(A-M_2)}$

$$E\left[Y\left(A,M_1(0),M_2(A,M_1(0))\right)-Y\left(A,M_1(0),M_2(0,M_1(0))\right)\right]$$
$$-E\left[Y\left(0,M_1(0),M_2(A,M_1(0))\right)-Y\left(0,M_1(0),M_2(0,M_1(0))\right)\right]$$

$$=E\left[Y\left(A,0,M_2(A,0)\right)-Y\left(A,0,M_2(0,0)\right)\right]$$
$$-E\left[Y\left(0,0,M_2(A,0)\right)-Y\left(0,0,M_2(0,0)\right)\right](PIE_{effect19})$$

$$+\left\{E\left[Y\left(A,0,M_2(A,M_1(0))\right)-Y\left(A,0,M_2(0,M_1(0))\right)\right]\right.$$
$$\left.-E\left[Y\left(A,0,M_2(A,0)\right)-Y\left(A,0,M_2(0,0)\right)\right]\right\}$$

$$-\left\{E\left[Y\left(A,M_1(0),M_2(A,M_1(0))\right)-Y\left(0,0,M_2(0,M_1(0))\right)\right]\right.$$
$$\left.-E\left[Y\left(0,0,M_2(A,0)\right)-Y\left(0,0,M_2(0,0)\right)\right]\right\}(PIE_{effect24})$$

$$+\left\{E\left[Y\left(A,M_1(0),M_2(A,M_1(0))\right)-Y\left(A,M_1(0),M_2(0,M_1(0))\right)\right]\right.$$
$$-E\left[Y\left(A,M_1(0),M_2(A,0)\right)-Y\left(A,M_1(0),M_2(0,0)\right)\right]$$
$$-E\left[Y\left(A,0,M_2(A,M_1(0))\right)-Y\left(A,0,M_2(0,M_1(0))\right)\right]$$
$$\left.+E\left[Y\left(0,0,M_2(A,0)\right)-Y\left(0,0,M_2(0,0)\right)\right]\right\}$$

$$-\left\{E\left[Y\left(0,M_1(0),M_2(A,M_1(0))\right)-Y\left(0,M_1(0),M_2(0,M_1(0))\right)\right]\right.$$
$$-E\left[Y\left(0,M_1(0),M_2(A,0)\right)-Y\left(0,M_1(0),M_2(0,0)\right)\right]$$
$$-E\left[Y\left(0,0,M_2(A,M_1(0))\right)-Y\left(0,0,M_2(0,M_1(0))\right)\right]$$
$$\left.+E\left[Y\left(0,0,M_2(A,0)\right)-Y\left(0,0,M_2(0,0)\right)\right]\right\}(PIE_{effect27})$$

$$\left\{E\left[Y\left(A,M_1(0),M_2(A,0)\right)-Y\left(0,M_1(0),M_2(A,0)\right)\right]\right.$$
$$\left.-E\left[Y\left(A,0,M_2(A,0)\right)-Y\left(A,0,M_2(0,0)\right)\right]\right\}$$

$$-\left\{E\left[Y\left(A,M_1(0),M_2(0,0)\right)-Y\left(0,M_1(0),M_2(0,0)\right)\right]\right.$$
$$\left.-E\left[Y\left(0,0,M_2(A,0)\right)-Y\left(0,0,M_2(0,0)\right)\right]\right\}(PIE_{effect28})$$

$\boldsymbol{INT_{med}(M_1 - M_2)}$

$$E[Y(0, M_1, M_2(A, M_1)) - Y(0, M_1, M_2(0, M_1))] - E\left[Y\left(0, M_1(0), M_2(A, M_1(0))\right) - Y\left(0, M_1(0), M_2(0, M_1(0))\right)\right]$$

$$= E[Y(0,0, M_2(A, M_1)) - Y(0,0, M_2(0, M_1))] - E\left[Y\left(0,0, M_2(A, M_1(0))\right) - Y\left(0,0, M_2(0, M_1(0))\right)\right] (PIE_{effect13})$$

$$+E[Y(0, M_1, M_2(A, 0)) - Y(0, M_1(0), M_2(A, 0))] - E[Y(0, M_1, M_2(0,0)) - Y(0, M_1(0), M_2(0,0))](PIE_{effect14})$$

$$+\{E[Y(0, M_1, M_2(A, M_1)) - Y(0, M_1, M_2(0, M_1))] - E[Y(0, M_1, M_2(A, 0)) - Y(0, M_1(0), M_2(A, 0))] - E[Y(0,0, M_2(A, M_1)) - Y(0,0, M_2(0, M_1))]\}$$

$$-\left\{E\left[Y\left(0, M(0), M_2(A, M_1(0))\right) - Y\left(0, M_1(0), M_2(0, M_1(0))\right)\right] - E[Y(0, M_1, M_2(0,0)) - Y(0, M_1(0), M_2(0,0))] - E\left[Y\left(0,0, M_2(A, M_1(0))\right) - Y\left(0,0, M_2(0, M_1(0))\right)\right]\right\} (PIE_{effect17})$$

$\boldsymbol{INT_{med}(A - M_1 - M_2)}$

$$\Big\{E\big[Y - Y\big(A, M_1, M_2(0, M_1)\big)\big] - E\Big[Y\Big(A, M_1(0), M_2\big(A, M_1(0)\big)\Big) - Y\Big(A, M_1(0), M_2\big(0, M_1(0)\big)\Big)\Big]\Big\}$$

$$-\Big\{E\big[Y\big(0, M_1, M_2(A, M_1)\big) - Y\big(0, M_1, M_2(0, M_1)\big)\big] - E\Big[Y\Big(0, M_1(0), M_2\big(A, M_1(0)\big)\Big) - Y\Big(0, M_1(0), M_2\big(0, M_1(0)\big)\Big)\Big]\Big\}$$

$$= \Big\{E\Big[Y\big(A, 0, M_2(A, M_1)\big) - Y\Big(A, 0, M_2\big(A, M_1(0)\big)\Big)\Big] - E\Big[Y\big(A, 0, M_2(0, M_1)\big) - Y\Big(A, 0, M_2\big(0, M_1(0)\big)\Big)\Big]\Big\}$$

$$-\Big\{E\Big[Y\big(0,0, M_2(A, M_1)\big) - Y\Big(0,0, M_2\big(A, M_1(0)\big)\Big)\Big] - E\Big[Y\big(0,0, M_2(0, M_1)\big) - Y\Big(0,0, M_2\big(0, M_1(0)\big)\Big)\Big]\Big\}(PIE_{effect23})$$

$$+\{E\big[Y\big(A, M_1, M_2(A, 0)\big) - Y\big(A, M_1(0), M_2(A, 0)\big)\big] - E\big[Y\big(A, M_1, M_2(0,0)\big) - Y\big(A, M_1(0), M_2(0,0)\big)\big]\} - \{E\big[Y\big(0, M_1, M_2(A, 0)\big) - Y\big(0, M_1(0), M_2(A, 0)\big)\big] - E\big[Y\big(0, M_1, M_2(0,0)\big) - Y\big(0, M_1(0), M_2(0,0)\big)\big]\}(PIE_{effect25})$$

$$
\begin{aligned}
&+\Big\{E\big[Y\big(A,M_1,M_2(A,M_1)\big)-Y\big(A,M_1,M_2(0,M_1)\big)\big]\\
&\qquad -E\big[Y\big(A,M_1,M_2(A,0)\big)-Y\big(A,M_1,M_2(0,0)\big)\big]\\
&\qquad -E\Big[Y\Big(A,M_1(0),M_2\big(A,M_1(0)\big)\Big)-Y\Big(A,M_1(0),M_2\big(0,M_1(0)\big)\Big)\Big]\\
&\qquad +E\big[Y\big(A,M_1(0),M_2(A,0)\big)-Y\big(A,M_1(0),M_2(0,0)\big)\big]\\
&\qquad -E\Big[Y\big(A,0,M_2(A,M_1)\big)-Y\Big(A,0,M_2\big(A,M_1(0)\big)\Big)\Big]\\
&\qquad +E\Big[Y\big(A,0,M_2(0,M_1)\big)-Y\Big(A,0,M_2\big(0,M_1(0)\big)\Big)\Big]\Big\}\\
&\qquad -\Big\{E\big[Y\big(0,M_1,M_2(A,M_1)\big)-Y\big(0,M_1,M_2(0,M_1)\big)\big]\\
&\qquad -E\big[Y\big(0,M_1,M_2(A,0)\big)-Y\big(0,M_1,M_2(0,0)\big)\big]\\
&\qquad -E\Big[Y\Big(0,M_1(0),M_2\big(A,M_1(0)\big)\Big)-Y\Big(0,M_1(0),M_2\big(0,M_1(0)\big)\Big)\Big]\\
&\qquad +E\big[Y\big(0,M_1(0),M_2(A,0)\big)-Y\big(0,M_1(0),M_2(0,0)\big)\big]\\
&\qquad -E\Big[Y\big(0,0,M_2(A,M_1)\big)-Y\Big(0,0,M_2\big(A,M_1(0)\big)\Big)\Big]\\
&\qquad +E\Big[Y\big(0,0,M_2(0,M_1)\big)-Y\Big(0,0,M_2\big(0,M_1(0)\big)\Big)\Big]\Big\}\,(PIE_{effect26})
\end{aligned}
$$

CEM

$$E\left[Y\left(0,M_1(0),M_2\left(0,M_1(0)\right)\right)-Y(0,0,0)\right]$$

$$= E\{[Y(0,1,1)-Y(0,1,0)-Y(0,0,1)+Y(0,0,0)]M_1(0)[M_2(0,1) - M_2(0,0)]\}(PIE_{effect29})$$

$$+E\{[Y(0,1,1)-Y(0,1,0)-Y(0,0,1)+Y(0,0,0)]M_1(0)M_2(0,0)\}(PIE_{effect30})$$

$$+E\{[Y(0,0,1)-Y(0,0,0)]M_1(0)[M_2(0,1)-M_2(0,0)]\}\left(PIE_{effect31}\right)$$

$$+E\{[Y(0,1,0)-Y(0,0,0)]M_1(0)\}\left(PIE_{effect32}\right)$$

$$+E\{[Y(0,0,1)-Y(0,0,0)]M_2(0,0)\}(PIE_{effect33})$$

6.2 因果參數辨識過程

$\boldsymbol{\phi_1}$

$E[Y(a, m_1, m_2)]$

$= E[Y(a, m_1, m_2)|A = a, M_1 = m_1, M_2 = m_2]$

$= E[Y|A = a, M_1 = m_1, M_2 = m_2]$

$\boldsymbol{\phi_2}$

$E\left[Y\left(a, m_1, M_2(e_2, m_1')\right)\right]$

$= \Sigma_{a,m_1,m_2} E\left[Y\left(a, m_1, M_2(e_2, m_1')\right)|A = a, M_1 = m_1, M_2(e_2, m_1') = m_2\right]$

$P(M_2(e_2, m_1') = m_2|A = a, M_1 = m_1)P(M_1 = m_1|A = a)P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y(a, m_1, m_2)|A = a, M_1 = m_1, M_2(e_2, m_1') = m_2]$

$P(M_2(e_2, m_1') = m_2|A = e_2, M_1 = m_1')P(M_1 = m_1|A = a)P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y|A = a, M_1 = m_1, M_2 = m_2]$

$P(M_2 = m_2|A = e_2, M_1 = m_1')P(M_1 = m_1|A = a)P(A = a)$

$= \Sigma_{m_2} E[Y|A = a, M_1 = m_1, M_2 = m_2]P(M_2 = m_2|A = e_2, M_1 = m_1')$

$\boldsymbol{\phi_3}$

$$E\left[Y\left(a, m_1, M_2\big(e_2, M_1(e_3)\big)\right)\right]$$

$$= \Sigma_a E\left[Y\left(a, m_1, M_2\big(e_2, M_1(e_3)\big)\right)|A = a\right] P(A = a)$$

$$= \Sigma_{a,m_1} E\big[Y\big(a, m_1, M_2(e_2, m_1)\big)|A = a, M_1(e_3) = m_1\big] P(M_1(e_3) = m_1|A = a)P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E\big[Y\big(a, m_1, M_2(e_2, m_1)\big)|A = a, M_1(e_3) = m_1, M_2(e_2, m_1) = m_2\big]$$

$$P(M_2(e_2, m_1) = m_2|A = a, M_1(e_3) = m_1)P(M_1(e_3) = m_1|A = a)P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y(a, m_1, m_2)|A = a, M_1 = m_1, M_2(e_2, m_1) = m_2]$$

$$P(M_2(e_2, m_1) = m_2|A = e_2, M_1 = m_1)P(M_1(e_3) = m_1|A = e_3)P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y|A = a, M_1 = m_1, M_2 = m_2]P(M_2 = m_2|A = e_2, M_1 = m_1)$$

$$P(M_1 = m_1|A = e_3)P(A = a)$$

$$= \Sigma_{m_1,m_2} E[Y|A = a, M_1 = m_1, M_2 = m_2]P(M_2 = m_2|A = e_2, M_1 = m_1)P(M_1 = m_1|A = e_3)$$

$\boldsymbol{\phi_4}$

$$E[Y(a, M_1(e_1), m_2)]$$

$$= \Sigma_{a,m_1,m_2} E[Y(a, M_1(e_1), m_2)|A = a, M_1(e_1) = m_1, M_2 = m_2]$$

$$P(M_2 = m_2|A = a, M_1(e_1) = m_1)P(M_1(e_1) = m_1|A = a)P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y(a, m_1, m_2)|A = a, M_1 = m_1, M_2 = m_2]$$

$$P(M_2 = m_2|A = a, M_1 = m_1)P(M_1 = m_1|A = e_1)P(A = a)$$

$$= \Sigma_{m_1} E[Y|A = a, M_1 = m_1, M_2 = m_2]P(M_1 = m_1|A = e_1)$$

$\boldsymbol{\phi_5}$

$E\big[Y\big(a, M_1(e_1), M_2(e_2, m_1)\big)\big]$

$= \Sigma_{a,m_1,m_2} E\big[Y\big(a, M_1(e_1), M_2(e_2, m_1)\big) | A = a, M_1(e_1) = m_1, M_2(e_2, m_1) = m_2\big]$

$P(M_2(e_2, m_1) = m_2 | A = a, M_1(e_1) = m_1) P(M(e_1) = m_1 | A = a) P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y(a, m_1, m_2) | A = a, M_1 = m_1, M_2 = m_2]$

$P(M_2 = m_2 | A = e_2, M_1 = m_1) P(M = m_1 | A = e_1) P(A = a)$

$= \Sigma_{m_1,m_2} E[Y | A = a, M_1 = m_1, M_2 = m_2] P(M_2 = m_2 | A = e_2, M_1 = m_1)$

$$P(M = m_1 | A = e_1)$$

$\boldsymbol{\phi_6}$

$E\left[Y\left(a, M_1(e_1), M_2\big(e_2, M_1(e_1)\big)\right)\right]$

$= \Sigma_{a,m_1,m_2} E\big[Y\big(a, M_1(e_1), M_2(e_2, m_1)\big) | A = a, M_1(e_1) = m_1, M_2(e_2, m_1) = m_2\big]$

$P(M_2(e_2, m_1) = m_2 | A = a, M_1(e_1) = m_1) P(M(e_1) = m_1 | A = a) P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y(a, m_1, m_2) | A = a, M_1 = m_1, M_2 = m_2]$

$P(M_2(e_2, m_1) = m_2 | A = e_2, M_1 = m_1) P(M = m_1 | A = e_1) P(A = a)$

$= \Sigma_{m_1,m_2} E[Y | A = a, M_1 = m_1, M_2 = m_2] P(M_2 = m_2 | A = e_2, M_1 = m_1)$

$$P(M = m_1 | A = e_1)$$

$\boldsymbol{\phi_7}$

$E[Y(A, m_1', m_2')]$

$= \Sigma_a E[Y(a, m_1', m_2')|A = a]P(A = a)$

$= \Sigma_a E[Y(a, m_1', m_2')|A = a, M_1 = m_1', M_2 = m_2']P(A = a)$

$= \Sigma_a E[Y|A = a, M_1 = m_1', M_2 = m_2']P(A = a)$

$\boldsymbol{\phi_8}$

$E\left[Y\left(A, m_1', M_2(e_2', m_1')\right)\right]$

$= \Sigma_{a,m_1,m_2} E\left[Y\left(A, m_1', M_2(e_2', m_1')\right)|A = a, M_1 = m_1, M_2(e_2', m_1') = m_2\right]$

$P(M_2(e_2', m_1') = m_2|A = a, M_1 = m_1)P(M_1 = m_1|A = a)P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y(a, m_1', m_2)|A = a, M_1 = m_1', M_2 = m_2]$

$P(M_2(e_2', m_1') = m_2|A = e_2', M_1 = m_1')P(M_1 = m_1|A = a)P(A = a)$

$= \Sigma_{a,m_2} E[Y|A = a, M_1 = m_1', M_2 = m_2]P(M_2 = m_2|A = e_2', M_1 = m_1')P(A = a)$

$\boldsymbol{\phi_9}$

$E\left[Y\left(a', m_1', M_2(A, m_1')\right)\right]$

$= \Sigma_a E\left[Y\left(a', m_1', M_2(a, m_1')\right)|A = a\right]P(A = a)$

$= \Sigma_{a,m_1,m_2} E\left[Y\left(a', m_1', M_2(a, m_1')\right)|A = a, M_1 = m_1, M_2(a, m_1') = m_2\right]$

$P(M_2(a, m_1') = m_2|A = a, M_1 = m_1)P(M_1 = m_1|A = a)P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y(a', m_1', m_2)|A = a', M_1 = m_1', M_2 = m_2]$

$P(M_2(a, m_1') = m_2|A = a, M_1 = m_1')P(M_1 = m_1|A = a)P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y|A = a', M_1 = m_1', M_2 = m_2]$

$P(M_2 = m_2|A = a, M_1 = m_1')P(M_1 = m_1|A = a)P(A = a)$

$= \Sigma_{a,m_2} E[Y|A = a', M_1 = m_1', M_2 = m_2]P(M_2 = m_2|A = a, M_1 = m_1')P(A = a)$

$\boldsymbol{\phi_{10}}$

$$E\left[Y\left(A, m_1', M_2(A, m_1')\right)\right]$$

$$= \Sigma_{a,m_1,m_2} E\left[Y\left(a, m_1', M_2(a, m_1')\right) | A = a, M_1 = m_1, M_2(a, m_1') = m_2\right]$$

$$P(M_2(a, m_1') = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y(a, m_1', m_2) | A = a, M_1 = m_1', M_2 = m_2]$$

$$P(M_2 = m_2 | A = a, M_1 = m_1') P(M_1 = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_2} E[Y | A = a, M_1 = m_1', M_2 = m_2] P(M_2 = m_2 | A = a, M_1 = m_1') P(A = a)$$

$\boldsymbol{\phi_{11}}$

$$E\left[Y\left(a', m_1', M_2(e_2', M_1)\right)\right]$$

$$= \Sigma_{a,m_1,m_2} E\left[Y\left(a', m_1', M_2(e_2', m_1)\right) | A = a, M_1 = m_1, M_2(e_2', m_1) = m_2\right]$$

$$P(M_2(e_2', m_1) = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y(a', m_1', m_2) | A = a', M_1 = m_1', M_2 = m_2]$$

$$P(M_2(e_2', m_1) = m_2 | A = e_2', M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y | A = a', M_1 = m_1', M_2 = m_2]$$

$$P(M_2 = m_2 | A = e_2', M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$$

$\boldsymbol{\phi_{12}}$

$$E\left[Y\left(A, m_1', M_2(e_2', M_1)\right)\right]$$

$$= \Sigma_{a,m_1,m_2} E\left[Y\left(a, m_1', M_2(e_2', m_1)\right) | A = a, M_1 = m_1, M_2(e_2', m_1) = m_2\right]$$

$$P(M_2(e_2', m_1) = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y(a, m_1', m_2) | A = a, M_1 = m_1', M_2 = m_2]$$

$$P(M_2 = m_2 | A = e_2', M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$$

$\boldsymbol{\phi_{13}}$

$$E\left[Y\left(a', m_1', M_2(A, M_1)\right)\right]$$

$$= \Sigma_{a,m_1,m_2} E\left[Y\left(a', m_1', M_2(A, M_1)\right) | A = a, M_1 = m_1, M_2(A, M_1) = m_2\right]$$

$$P(M_2(A, M_1) = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E\left[Y\left(a', m_1', M_2(a, m_1)\right) | A = a, M_1 = m_1, M_2(a, m_1) = m_2\right]$$

$$P(M_2(a, m_1) = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y(a', m_1', m_2) | A = a', M_1 = m_1', M_2 = m_2]$$

$$P(M_2(a, m_1) = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y | A = a', M_1 = m_1', M_2 = m_2]$$

$$P(M_2 = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$$

$\boldsymbol{\phi_{14}}$

$E\left[Y\left(A, m_1', M_2(A, M_1)\right)\right]$

$= \Sigma_{a,m_1,m_2} E\left[Y\left(A, m_1', M_2(A, M_1)\right) | A = a, M_1 = m_1, M_2(A, M_1) = m_2\right]$

$$P(M_2(A, M_1) = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$$

$= \Sigma_{a,m_1,m_2} E\left[Y\left(a, m_1', M_2(a, m_1)\right) | A = a, M_1 = m_1, M_2(a, m_1) = m_2\right]$

$$P(M_2(a, m_1) = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$$

$= \Sigma_{a,m_1,m_2} E[Y(a, m_1', m_2) | A = a, M_1 = m_1', M_2 = m_2]$

$$P(M_2 = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$$

$= \Sigma_{a,m_1,m_2} E[Y | A = a, M_1 = m_1', M_2 = m_2]$

$$P(M_2 = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$$

$\boldsymbol{\phi_{15}}$

$E\left[Y\left(A, m_1', M_2(e_2', M(e_1'))\right)\right]$

$= \Sigma_{a,m_1,m_2} E\left[Y\left(A, m_1', M_2(e_2', M(e_1'))\right) | A = a, M(e_1') = m_1, M_2(e_2', M(e_1')) = m_2\right]$

$$P(M_2(e_2', M(e_1')) = m_2 | A = a, M(e_1') = m_1) P(M(e_1') = m_1 | A = a) P(A = a)$$

$= \Sigma_{a,m_1,m_2} E\left[Y\left(A, m_1', M_2(e_2', m_1)\right) | A = a, M(e_1') = m_1, M_2(e_2', m_1) = m_2\right]$

$$P(M_2(e_2', m_1) = m_2 | A = a, M(e_1') = m_1) P(M(e_1') = m_1 | A = a) P(A = a)$$

$= \Sigma_{a,m_1,m_2} E[Y(a, m_1', m_2) | A = a, M(e_1') = m_1, M_2(e_2', m_1) = m_2]$

$$P(M_2(e_2', m_1) = m_2 | A = e_2', M_1 = m_1) P(M(e_1') = m_1 | A = e_1') P(A = a)$$

$= \Sigma_{a,m_1,m_2} E[Y | A = a, M_1 = m_1', M_2 = m_2] P(M_2 = m_2 | A = e_2', M_1 = m_1)$

$$P(M_1 = m_1 | A = e_1') P(A = a)$$

$\boldsymbol{\phi_{16}}$

$$E\left[Y\left(a', m_1', M_2\left(A, M_1(e_1')\right)\right)\right]$$

$$= \Sigma_{a,m_1,m_2} E\left[Y\left(a', m_1', M_2(A, M(e_1'))\right) | A = a, M(e_1') = m_1, M_2(A, M(e_1')) = m_2\right]$$

$$P(M_2(A, M(e_1')) = m_2 | A = a, M(e_1') = m_1) P(M(e_1') = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E\left[Y\left(a', m_1', M_2(A, M(e_1'))\right) | A = a, M(e_1') = m_1, M_2(a, m_1) = m_2\right]$$

$$P(M_2(a, M(e_1')) = m_2 | A = a, M(e_1') = m_1) P(M(e_1') = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E\left[Y\left(a', m_1', M_2(a, m_1)\right) | A = a, M(e_1') = m_1, M_2(a, m_1) = m_2\right]$$

$$P(M_2(a, m_1) = m_2 | A = a, M(e_1') = m_1) P(M(e_1') = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y(a', m_1', m_2) | A = a, M(e_1') = m_1, M_2(a, m_1) = m_2]$$

$$P(M_2(a, m_1) = m_2 | A = a, M_1 = m_1) P(M(e_1') = m_1 | A = e_1') P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y(a', m_1', m_2) | A = a', M_1 = m_1', M_2 = m_2]$$

$$P(M_2 = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = e_1') P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y | A = a', M_1 = m_1', M_2 = m_2] P(M_2 = m_2 | A = a, M_1 = m_1)$$

$$P(M_1 = m_1 | A = e_1') P(A = a)$$

$\boldsymbol{\phi_{17}}$

$E\left[Y\left(A, m_1', M_2\left(A, M_1(e_1')\right)\right)\right]$

$= \Sigma_{a,m_1,m_2} E\left[Y\left(A, m_1', M_2(A, M(e_1'))\right) | A = a, M(e_1') = m_1, M_2(A, M(e_1')) = m_2\right]$

$P(M_2(A, M(e_1')) = m_2 | A = a, M(e_1') = m_1) P(M(e_1') = m_1 | A = a) P(A = a)$

$= \Sigma_{a,m_1,m_2} E\left[Y\left(a, m_1', M_2(a, m_1)\right) | A = a, M(e_1') = m_1, M_2(a, m_1) = m_2\right]$

$P(M_2(a, m_1) = m_2 | A = a, M(e_1') = m_1) P(M(e_1') = m_1 | A = a) P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y(a, m_1', m_2) | A = a, M(e_1') = m_1, M_2(a, m_1) = m_2]$

$P(M_2(a, m_1) = m_2 | A = a, M(e_1') = m_1) P(M(e_1') = m_1 | A = a) P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y(a, m_1', m_2) | A = a, M_1 = m_1', M_2 = m_2]$

$P(M_2 = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = e_1') P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y | A = a, M_1 = m_1', M_2 = m_2]$

$P(M_2 = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = e_1') P(A = a)$

$\boldsymbol{\phi_{18}}$

$E[Y(A, M_1, m_2')]$

$= \Sigma_{a,m_1,m_2} E[Y(A, M_1, m_2') | A = a, M_1 = m_1, M_2 = m_2]$

$P(M_2 = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y(a, m_1, m_2') | A = a, M_1 = m_1, M_2 = m_2']$

$P(M_2 = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = e_1) P(A = a)$

$= \Sigma_{a,m_1} E[Y | A = a, M_1 = m_1, M_2 = m_2'] P(M_1 = m_1 | A = a) P(A = a)$

$\boldsymbol{\phi_{19}}$

$E[Y(a', M_1, m_2')]$

$= \Sigma_{a,m_1,m_2} E[Y(a', M_1, m_2')|A = a, M_1 = m_1, M_2 = m_2]$

$P(M_2 = m_2|A = a, M_1 = m_1)P(M_1 = m_1|A = a)P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y(a', m_1, m_2')|A = a, M_1 = m_1, M_2 = m_2]$

$P(M_2 = m_2|A = a, M_1 = m_1)P(M_1 = m_1|A = a)P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y(a', m_1, m_2')|A = a', M_1 = m_1, M_2 = m_2']$

$P(M_2 = m_2|A = a, M_1 = m_1)P(M_1 = m_1|A = a)P(A = a)$

$= \Sigma_{a,m_1} E[Y|A = a', M_1 = m_1, M_2 = m_2']P(M_1 = m_1|A = a)P(A = a)$

$\boldsymbol{\phi_{20}}$

$E[Y(A, M_1(e_1'), m_2')]$

$= \Sigma_{a,m_1,m_2} E[Y(A, M_1(e_1'), m_2')|A = a, M_1(e_1') = m_1, M_2 = m_2]$

$P(M_2 = m_2|A = a, M_1(e_1') = m_1)P(M_1(e_1') = m_1|A = a)P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y(a, m_1, m_2')|A = a, M_1 = m_1, M_2 = m_2']$

$P(M_2 = m_2|A = a, M_1 = m_1)P(M_1 = m_1|A = e_1')P(A = a)$

$= \Sigma_{a,m_1} E[Y|A = a, M_1 = m_1, M_2 = m_2']P(M_1 = m_1|A = e_1')P(A = a)$

$\boldsymbol{\phi_{21}}$

$$E\left[Y\left(a', M_1, M_2(e_2', m_1')\right)\right]$$

$$= \Sigma_{a,m_1,m_2} E\left[Y\left(a', M_1, M_2(e_2', m_1')\right) \middle| A = a, M_1 = m_1, M_2(e_2', m_1') = m_2\right]$$

$$P(M_2(e_2', m_1') = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E\left[Y\left(a', m_1, M_2(e_2', m_1')\right) \middle| A = a, M_1 = m_1, M_2(e_2', m_1') = m_2\right]$$

$$P(M_2(e_2', m_1') = m_2 | A = e_2', M_1 = m_1') P(M_1 = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y(a', m_1, m_2) | A = a, M_1 = m_1, M_2 = m_2]$$

$$P(M_2(e_2', m_1') = m_2 | A = e_2', M_1 = m_1') P(M_1 = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y | A = a', M_1 = m_1, M_2 = m_2]$$

$$P(M_2 = m_2 | A = e_2', M_1 = m_1') P(M_1 = m_1 | A = a) P(A = a)$$

$\boldsymbol{\phi_{22}}$

$$E\left[Y\left(A, M_1, M_2(e_2', m_1')\right)\right]$$

$$= \Sigma_{a,m_1,m_2} E\left[Y\left(A, M_1, M_2(e_2', m_1')\right) \middle| A = a, M_1 = m_1, M_2(e_2', m_1') = m_2\right]$$

$$P(M_2(e_2', m_1') = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y(a, m_1, m_2) | A = a, M_1 = m_1, M_2(e_2', m_1') = m_2]$$

$$P(M_2(e_2', m_1') = m_2 | A = e_2', M_1 = m_1') P(M_1 = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y | A = a, M_1 = m_1, M_2 = m_2]$$

$$P(M_2 = m_2 | A = e_2', M_1 = m_1') P(M_1 = m_1 | A = a) P(A = a)$$

$\boldsymbol{\phi_{23}}$

$E\left[Y\left(a', M_1, M_2(A, m_1')\right)\right]$

$= \Sigma_{a,m_1,m_2} E\left[Y\left(a', M_1, M_2(A, m_1')\right) \middle| \, A = a, M_1 = m_1, M_2(A, m_1') = m_2\right]$

$$P(M_2(A, m_1') = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$$

$= \Sigma_{a,m_1,m_2} E\left[Y\left(a', M_1, M_2(a, m_1')\right) \middle| \, A = a, M_1 = m_1, M_2(a, m_1') = m_2\right]$

$$P(M_2(a, m_1') = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$$

$= \Sigma_{a,m_1,m_2} E\left[Y\left(a', m_1, M_2(a, m_1')\right) \middle| \, A = a, M_1 = m_1, M_2(a, m_1') = m_2\right]$

$$P(M_2(a, m_1') = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$$

$= \Sigma_{a,m_1,m_2} E[Y(a', m_1, m_2) | \, A = a', M_1 = m_1, M_2 = m_2]$

$$P(M_2(a, m_1') = m_2 | A = a, M_1 = m_1') P(M_1 = m_1 | A = a) P(A = a)$$

$= \Sigma_{a,m_1,m_2} E[Y | \, A = a', M_1 = m_1, M_2 = m_2]$

$$P(M_2 = m_2 | A = a, M_1 = m_1') P(M_1 = m_1 | A = a) P(A = a)$$

$\boldsymbol{\phi_{24}}$

$E\left[Y\left(A, M_1, M_2(A, m_1')\right)\right]$

$= \Sigma_{a,m_1,m_2} E\left[Y\left(A, M_1, M_2(A, m_1')\right) \middle| \, A = a, M_1 = m_1, M_2(A, m_1') = m_2\right]$

$$P(M_2(A, m_1') = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$$

$= \Sigma_{a,m_1,m_2} E\left[Y\left(a, M_1, M_2(a, m_1')\right) \middle| \, A = a, M_1 = m_1, M_2(a, m_1') = m_2\right]$

$$P(M_2(a, m_1') = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$$

$= \Sigma_{a,m_1,m_2} E[Y(a, m_1, m_2) | \, A = a, M_1 = m_1, M_2(a, m_1') = m_2]$

$$P(M_2(a, m_1') = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$$

$= \Sigma_{a,m_1,m_2} E[Y(a, m_1, m_2) | \, A = a, M_1 = m_1, M_2 = m_2]$

$$P(M_2(a, m_1') = m_2 | A = a, M_1 = m_1') P(M_1 = m_1 | A = a) P(A = a)$$

$= \Sigma_{a,m_1,m_2} E[Y | \, A = a, M_1 = m_1, M_2 = m_2] P(M_2 = m_2 | A = a, M_1 = m_1')$

$$P(M_1 = m_1 | A = a) P(A = a)$$

$\boldsymbol{\phi_{25}}$

$$E\big[Y\big(A, M_1(e_1'), M_2(e_2', m_1')\big)\big]$$

$$= \Sigma_{a,m_1,m_2} E\big[Y\big(A, M_1(e_1'), M_2(e_2', m_1')\big) |\ A = a, M_1(e_1') = m_1, M_2(e_2', m_1') = m_2\big]$$

$$P(M_2(e_2', m_1') = m_2 | A = a, M_1(e_1') = m_1) P(M_1(e_1') = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y(a, m_1, m_2) |\ A = a, M_1(e_1') = m_1, M_2(e_2', m_1') = m_2]$$

$$P(M_2(e_2', m_1') = m_2 | A = a, M_1 = m_1') P(M_1(e_1') = m_1 | A = e_1') P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y |\ A = a, M_1 = m_1, M_2 = m_2] P(M_2(e_2', m_1') = m_2 | A = e_2', M_1 = m_1')$$

$$P(M_1 = m_1 | A = e_1') P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y |\ A = a, M_1 = m_1, M_2 = m_2] P(M_2 = m_2 | A = e_2', M_1 = m_1')$$

$$P(M_1 = m_1 | A = e_1') P(A = a)$$

$\boldsymbol{\phi_{26}}$

$$E\big[Y\big(a', M_1(e_1'), M_2(A, m_1')\big)\big]$$

$$= \Sigma_{a,m_1,m_2} E\big[Y\big(a', M_1(e_1'), M_2(A, m_1')\big) |\ A = a, M_1(e_1') = m_1, M_2(A, m_1') = m_2\big]$$

$$P(M_2(A, m_1') = m_2 | A = a, M_1(e_1') = m_1) P(M_1(e_1') = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y(a', m_1, m_2) |\ A = a, M_1(e_1') = m_1, M_2(a, m_1') = m_2]$$

$$P(M_2(a, m_1') = m_2 | A = a, M_1(e_1') = m_1) P(M_1(e_1') = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y(a', m_1, m_2) |\ A = a', M_1 = m_1, M_2 = m_2]$$

$$P(M_2 = m_2 | A = a, M_1 = m_1') P(M_1(e_1') = m_1 | A = e_1') P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y |\ A = a', M_1 = m_1, M_2 = m_2] P(M_2 = m_2 | A = a, M_1 = m_1')$$

$$P(M_1 = m_1 | A = e_1') P(A = a)$$

$\boldsymbol{\phi_{27}}$

$E\left[Y\left(A, M_1(e_1'), M_2(A, m_1')\right)\right]$

$= \Sigma_{a,m_1,m_2} E\left[Y\left(A, M_1(e_1'), M_2(A, m_1')\right) \middle| A = a, M_1(e_1') = m_1, M_2(A, m_1') = m_2\right]$

$P(M_2(A, m_1') = m_2 | A = a, M_1(e_1') = m_1) P(M_1(e_1') = m_1 | A = a) P(A = a)$

$= \Sigma_{a,m_1,m_2} E\left[Y\left(a, m_1, M_2(a, m_1')\right) \middle| A = a, M_1(e_1') = m_1, M_2(a, m_1') = m_2\right]$

$P(M_2(a, m_1') = m_2 | A = a, M_1(e_1') = m_1) P(M_1(e_1') = m_1 | A = a) P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y(a, m_1, m_2) | A = a, M_1(e_1') = m_1, M_2(a, m_1') = m_2]$

$P(M_2(a, m_1') = m_2 | A = a, M_1(e_1') = m_1) P(M_1(e_1') = m_1 | A = e_1') P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y(a, m_1, m_2) | A = a, M_1 = m_1, M_2 = m_2]$

$P(M_2 = m_2 | A = a, M_1 = m_1') P(M_1 = m_1 | A = e_1') P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y | A = a, M_1 = m_1, M_2 = m_2] P(M_2 = m_2 | A = a, M_1 = m_1')$

$P(M_1 = m_1 | A = e_1') P(A = a)$

$\boldsymbol{\phi_{28}}$

$E\left[Y\left(A, M_1, M_2(e_2', M_1)\right)\right]$

$= \Sigma_{a,m_1,m_2} E\left[Y\left(A, M_1, M_2(e_2', M_1)\right) \middle| A = a, M_1 = m_1, M_2(e_2', M_1) = m_2\right]$

$P(M_2(e_2', M_1) = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$

$= \Sigma_{a,m_1,m_2} E\left[Y\left(a, m_1, M_2(e_2', m_1)\right) \middle| A = a, M_1 = m_1, M_2(e_2', m_1) = m_2\right]$

$P(M_2(e_2', m_1) = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y(a, m_1, m_2) | A = a, M_1 = m_1, M_2(e_2', m_1) = m_2]$

$P(M_2(e_2', m_1) = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y(a, m_1, m_2) | A = a, M_1 = m_1, M_2 = m_2]$

$P(M_2(e_2', m_1) = m_2 | A = e_2', M_1 = m_1) P(M_1 = m_1 | A = a) P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y | A = a, M_1 = m_1, M_2 = m_2] P(M_2 = m_2 | A = e_2', M_1 = m_1)$

$P(M_1 = m_1 | A = a) P(A = a)$

$\boldsymbol{\phi_{29}}$

$E[Y(a', M_1, M_2(e_2', M_1))]$

$= \Sigma_{a,m_1,m_2} E[Y(a', M_1, M_2(e_2', M_1))|\ A = a, M_1 = m_1, M_2(e_2', M_1) = m_2]$

$P(M_2(e_2', M_1) = m_2|A = a, M_1 = m_1)P(M_1 = m_1|A = a)P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y(a', m_1, M_2(e_2', m_1))|\ A = a, M_1 = m_1, M_2(e_2', m_1) = m_2]$

$P(M_2(e_2', m_1) = m_2|A = a, M_1 = m_1)P(M_1 = m_1|A = a)P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y(a', m_1, m_2)|\ A = a, M_1 = m_1, M_2(e_2', m_1) = m_2]$

$P(M_2(e_2', m_1) = m_2|A = a, M_1 = m_1)P(M_1 = m_1|A = a)P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y(a', m_1, m_2)|\ A = a', M_1 = m_1, M_2 = m_2]$

$P(M_2(e_2', m_1) = m_2|A = e_2', M_1 = m_1)P(M_1 = m_1|A = a)P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y|\ A = a', M_1 = m_1, M_2 = m_2]P(M_2 = m_2|A = e_2', M_1 = m_1)$

$P(M_1 = m_1|A = a)P(A = a)$

$\boldsymbol{\phi_{30}}$

$E[Y(a', M_1, M_2(A, M_1))]$

$= \Sigma_{a,m_1,m_2} E[Y(a', M_1, M_2(A, M_1))|\ A = a, M_1 = m_1, M_2(A, M_1) = m_2]$

$P(M_2(A, M_1) = m_2|A = a, M_1 = m_1)P(M_1 = m_1|A = a)P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y(a', M_1, M_2(a, m_1))|\ A = a, M_1 = m_1, M_2(a, m_1) = m_2]$

$P(M_2(a, m_1) = m_2|A = a, M_1 = m_1)P(M_1 = m_1|A = a)P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y(a', m_1, m_2)|\ A = a, M_1 = m_1, M_2(a, m_1) = m_2]$

$P(M_2(a, m_1) = m_2|A = a, M_1 = m_1)P(M_1 = m_1|A = a)P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y|\ A = a', M_1 = m_1, M_2(a, m_1) = m_2]$

$P(M_2(a, m_1) = m_2|A = a, M_1 = m_1)P(M_1 = m_1|A = a)P(A = a)$

$= \Sigma_{a,m_1,m_2} E[Y|\ A = a', M_1 = m_1, M_2 = m_2]P(M_2 = m_2|A = a, M_1 = m_1)$

$P(M_1 = m_1|A = a)P(A = a)$

$\boldsymbol{\phi_{31}}$

$$E\left[Y\left(A, M_1(e_1'), M_2\left(e_2', M_1(e_1')\right)\right)\right]$$

$$= \Sigma_{a,m_1,m_2} E\left[Y\left(A, M_1(e_1'), M_2\left(e_2', M_1(e_1')\right)\right) \mid A = a, M_1(e_1') = m_1, M_2\left(e_2', M_1(e_1')\right) = m_2\right] P\left(M_2\left(e_2', M_1(e_1')\right) = m_2 \mid A = a, M_1(e_1') = m_1\right)$$

$$P(M_1(e_1') = m_1 \mid A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E\left[Y\left(A, m_1, M_2(e_2', m_1)\right) \mid A = a, M_1(e_1') = m_1, M_2(e_2', m_1) = m_2\right]$$

$$P(M_2(e_2', m_1) = m_2 \mid A = a, M_1(e_1') = m_1) P(M_1(e_1') = m_1 \mid A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y(a, m_1, m_2) \mid A = a, M_1(e_1') = m_1, M_2(e_2', m_1) = m_2]$$

$$P(M_2(e_2', m_1) = m_2 \mid A = a, M_1 = m_1) P(M_1(e_1') = m_1 \mid A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y(a, m_1, m_2) \mid A = a, M_1 = m_1, M_2 = m_2]$$

$$P(M_2(e_2', m_1) = m_2 \mid A = e_2', M_1 = m_1) P(M_1(e_1') = m_1 \mid A = e_1') P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y \mid A = a, M_1 = m_1, M_2 = m_2] P(M_2 = m_2 \mid A = e_2', M_1 = m_1)$$

$$P(M_1 = m_1 \mid A = e_1') P(A = a)$$

$\boldsymbol{\phi_{32}}$

$$E\left[Y\left(a', M_1(e_1'), M_2\big(A, M_1(e_1')\big)\right)\right]$$

$$= \Sigma_{a,m_1,m_2} E\left[Y\left(a', M_1(e_1'), M_2\big(A, M_1(e_1')\big)\right) \mid A = a, M_1(e_1') = m_1, M_2\big(A, M_1(e_1')\big)\right.$$

$$\left. = m_2\right] P\big(M_2\big(A, M_1(e_1')\big) = m_2 | A = a, M_1(e_1') = m_1\big)$$

$$P(M_1(e_1') = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E\big[Y\big(a', m_1, M_2(a, m_1)\big) \mid A = a, M_1(e_1') = m_1, M_2(a, m_1) = m_2\big]$$

$$P(M_2(e_2', m_1) = m_2 | A = a, M_1(e_1') = m_1)$$

$$P(M_1(e_1') = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y(a', m_1, m_2) \mid A = a', M_1(e_1') = m_1, M_2(a, m_1) = m_2]$$

$$P(M_2(a, m_1) = m_2 | A = a, M_1(e_1') = m_1) P(M_1(e_1') = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y(a', m_1, m_2) \mid A = a', M_1 = m_1, M_2 = m_2]$$

$$P(M_2(a, m_1) = m_2 | A = a, M_1 = m_1) P(M_1(e_1') = m_1 | A = e_1') P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y \mid A = a', M_1 = m_1, M_2 = m_2] P(M_2 = m_2 | A = a, M_1 = m_1)$$

$$P(M_1 = m_1 | A = e_1') P(A = a)$$

$\boldsymbol{\phi_{33}}$

$$E\left[Y\left(A, M_1(e_1'), M_2\left(A, M_1(e_1')\right)\right)\right]$$

$$= \Sigma_{a,m_1,m_2} E\left[Y\left(A, M_1(e_1'), M_2\left(A, M_1(e_1')\right)\right) \mid A = a, M_1(e_1') = m_1, M_2\left(A, M_1(e_1')\right) = m_2\right] P\left(M_2\left(A, M_1(e_1')\right) = m_2 | A = a, M_1(e_1') = m_1\right) P(M_1(e_1') = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E\left[Y\left(a, M_1(e_1'), M_2\left(a, M_1(e_1')\right)\right) \mid A = a, M_1(e_1') = m_1, M_2\left(a, M_1(e_1')\right) = m_2\right] P\left(M_2\left(a, M_1(e_1')\right) = m_2 | A = a, M_1(e_1') = m_1\right) P(M_1(e_1') = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E\left[Y\left(a, m_1, M_2(a, m_1)\right) \mid A = a, M_1(e_1') = m_1, M_2(a, m_1) = m_2\right] P(M_2(a, m_1) = m_2 | A = a, M_1(e_1') = m_1) P(M_1(e_1') = m_1 | A = a) P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y(a, m_1, m_2) \mid A = a, M_1(e_1') = m_1, M_2(a, m_1) = m_2] P(M_2 = m_2 | A = a, M_1 = m_1) P(M_1(e_1') = m_1 | A = e_1') P(A = a)$$

$$= \Sigma_{a,m_1,m_2} E[Y \mid A = a, M_1 = m_1, M_2 = m_2] P(M_2 = m_2 | A = a, M_1 = m_1) P(M_1 = m_1 | A = e_1') P(A = a)$$